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16. Abstract <p>Most multipath assignment techniques are generated based on either path enumeration or path diversion. Path enumeration models primarily reiterate the assignment procedure with variable link impedance inputs. Burrell's algorithm is a typical path enumeration model in which the link impedances are assumed to be randomly distributed to account for errors in the driver's perception in link travel time. Path diversion models assign trips to alternate paths without repeating the assignment procedure. The most noted path diversion model is Dial's algorithm. Dial's technique originated from logit discrete choice theory in that each "reasonable" path between a particular O-D pair is assigned a portion of the trips according to a route-choice probability.</p> <p>The literature review indicates that these multiple path algorithms can be incorporated into the capacity-restraint process, either iterative or incrementa. Burrell's algorithm can be implemented either in a single-pass procedure or with the capacity-restraint procedure. Paths are enumerated by repeating simulations of link impedances for each origin zone (or a number of origin zones) in a single-pass procedure; paths are enumerated by repeating simulations of link impedances for each assignment stage when combined with the capacity-restraint procedure. In theory, Dial's algorithm can be implemented with the capacity-restraint procedure although his algorithm is a single-pass procedure.</p>					
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**MULTIPATH TRAFFIC ASSIGNMENT:
A REVIEW OF THE LITERATURE**

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The Texas A&M University System
College Station, Texas 77843

September 1990

METRIC (SI*) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH				
in	inches	2.54	centimetres	cm
ft	feet	0.3048	metres	m
yd	yards	0.914	metres	m
mi	miles	1.61	kilometres	km

AREA

in ²	square inches	645.2	centimetres squared	cm ²
ft ²	square feet	0.0929	metres squared	m ²
yd ²	square yards	0.836	metres squared	m ²
mi ²	square miles	2.59	kilometres squared	km ²
ac	acres	0.395	hectares	ha

MASS (weight)

oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams	Mg

VOLUME

fl oz	fluid ounces	29.57	millilitres	mL
gal	gallons	3.785	litres	L
ft ³	cubic feet	0.0328	metres cubed	m ³
yd ³	cubic yards	0.0765	metres cubed	m ³

NOTE: Volumes greater than 1000 L shall be shown in m³.

TEMPERATURE (exact)

°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C
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APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH				
mm	millimetres	0.039	inches	in
m	metres	3.28	feet	ft
m	metres	1.09	yards	yd
km	kilometres	0.621	miles	mi

AREA

mm ²	millimetres squared	0.0016	square inches	in ²
m ²	metres squared	10.764	square feet	ft ²
km ²	kilometres squared	0.39	square miles	mi ²
ha	hectares (10 000 m ²)	2.53	acres	ac

MASS (weight)

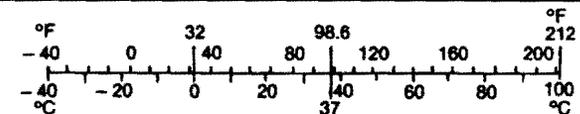
g	grams	0.0353	ounces	oz
kg	kilograms	2.205	pounds	lb
Mg	megagrams (1 000 kg)	1.103	short tons	T

VOLUME

mL	millilitres	0.034	fluid ounces	fl oz
L	litres	0.264	gallons	gal
m ³	metres cubed	35.315	cubic feet	ft ³
m ³	metres cubed	1.308	cubic yards	yd ³

TEMPERATURE (exact)

°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F
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These factors conform to the requirement of FHWA Order 5190.1A.

* SI is the symbol for the International System of Measurements

ABSTRACT

This report is a review and analysis of the existing multiple path assignment techniques. Multipath (stochastic) assignment techniques were developed in recognition of one or more of the following:

1. The impedance (travel time) on a link, or series of links, is a random variable and not a fixed value.
2. For most zone pairs, there are a variety of paths for which the travel times are similar.
3. Drivers do not, and cannot, differentiate between paths for which the travel time (impedance) differs by a small amount.
4. Where drivers have a multiplicity of choices, the specific route taken will vary depending upon conditions encountered on a given trip.

Most multipath assignment techniques are generated based on either path enumeration or path diversion. Path enumeration models primarily reiterate the assignment procedure with variable link impedance inputs. Burrell's algorithm is a typical path enumeration model in which the link impedances are assumed to be randomly distributed to account for errors in the driver's perception in link travel time. Path diversion models assign trips to alternate paths without repeating the assignment procedure. The most noted path diversion model is Dial's algorithm. Dial's technique originated from logit discrete choice theory in that each "reasonable" path between a particular O-D pair is assigned a portion of the trips according to a route-choice probability.

The literature review indicates that these multiple path algorithms can be incorporated into the capacity-restraint process, either iterative or incremental procedure. Burrell's algorithm can be implemented either in a single-pass procedure or with the capacity-restraint procedure. Paths are enumerated by repeating simulations of link impedances for each origin zone (or a number of origin zones) in a single-pass procedure; paths are enumerated by repeating simulations of link impedances for each assignment stage when combined with the capacity-restraint procedure. In theory, Dial's algorithm can be implemented with the capacity-restraint procedure although his algorithm is a single-pass procedure.

DISCLAIMER

The contents of this report reflect the views of the authors who are responsible for the opinions, findings, and conclusions presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration or the State Department of Highways and Public Transportation. This report does not constitute a standard, specification, or regulation.

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I. INTRODUCTION

STUDY PURPOSE

Traffic assignment is the final stage of the traditional four-step urban transportation planning process (trip generation, trip distribution, modal split, and traffic assignment). In Texas, the preparation of traffic assignments (the product of the traffic assignment process) and the refinements to traffic assignments are the responsibility of the Transportation Systems Planning Section (D-10P). However, the traffic assignment process for a particular urban area may require staff participation from the city(s), the Metropolitan Planning Organization, the transit agency, the county(s), the toll authority, and the SDHPT District Office.

Highway traffic assignments are used for a variety of purposes. Frequent applications include project ranking, project development, analysis of highway alternatives, corridor analysis, geometric design, pavement design, and environmental analysis. The objective of this study is to improve the efficiency, effectiveness, and responsiveness of the traffic assignment process. One major element of the research study is to review and evaluate potential innovative traffic assignment techniques that might improve the effectiveness of traffic assignments.

Although various assignment techniques are available in practice, most of them use the all-or-nothing algorithm as a major path searching mechanism in their procedures. The basic all-or-nothing assignment algorithm assigns all the trips from an origin to a destination to a minimum time (impedance) path. In a detailed highway network, there may be several paths between a pair of centroids which are very close to that of the minimum path. In such cases, it is logical that all these paths should receive some portion of the trips. Further, it would seem logical that a multiple path searching mechanism, instead of a single path, be used in most assignment techniques.

Virtually all assignment models assign traffic based on one of the two following criteria:

1. Assignment of all trips between an origin and a destination to a unique optimum (i.e., minimum cost) route, or

2. Assignment to multiple cost routes (i.e., other routes in addition to the minimum cost route).

For example, iterative and incremental capacity-restraint procedures using successive applications of the all-or-nothing assignment algorithm fit into the first category, whereas Dial's Algorithm fits into the second.

From a behavioral point of view, the second criterion is unquestionably more realistic. Trip makers do not choose only the minimum cost route. When there are alternate routes having similar travel times between an origin and a destination, some traffic will use each route. Even utility-maximizing drivers sometimes use alternate paths due to the deficiency of perfect information of roadway conditions and estimation errors. Capacity-restraint assignment techniques attempt to consider operating conditions through a successive adjustment process in which link speeds are adjusted based upon the assigned volume to a coded capacity. The purpose of this report was to investigate theories related to 1) the multipath algorithms, 2) the capacity-restraint procedure, and 3) the incorporation of a multipath algorithm in the capacity-restraint procedure.

ORGANIZATION OF THIS REPORT

In this report, the assignment techniques used in current practice are reviewed. Although the report focuses on the multiple path assignment techniques, it is necessary to include the capacity-restraint procedure since it is the foundation of path enumeration models. The all-or-nothing capacity-restraint procedure are discussed in Chapter II together with an overview of associated problems.

Chapter III examines those methods which are referred to as multiple path procedures. They are classified into two groups: path enumeration models and path diversion models. The path enumeration models primarily reiterate the assignment procedure with different link impedance values; the path diversion models divert trips to alternate paths without repeating the assignment procedure. Capacity restraint procedures can be classified as path enumeration models; Burrell's procedure is also a path enumeration technique with link impedances randomly selected from a prescribed statistical distribution. The exemplary path diversion models are Dial's and Gunnarsson's algorithms.

The procedure of these models is summarized; their virtues and deficiencies are discussed; and the models are compared.

Chapter IV focuses on the applications of the multiple path models, especially on their applications to actual networks. Since there are several articles involved, the chapter is organized as a listing of articles with annotations. Finally, Chapter V is a summary of the literature review as well as the proposed future research direction.

II. CAPACITY-RESTRAINT PROCEDURE

TRAFFIC ASSIGNMENT PRACTICE AND PROBLEMS

Traffic assignment is the process of allocating a given number of interzonal trips to links in an existing or proposed urban transportation network. In general, the traffic assignment procedure requires the following inputs:

1. a selective description of the assignment network,
2. an origin-destination matrix (O-D table) of traffic demand, and
3. the associated link performance (or impedance) functions.

The end result of the procedure is simply an assigned volume on each of the network links. However, several measures of effectiveness can also be derived from the result, such as travel time, travel speed, vehicle-miles of travel, vehicle-hours of travel, etc. These measures are essential in detecting future deficiencies in the existing transportation system, in evaluating future improvements and additions to the transportation network, and in providing systematic and reproducible information for the evaluation of alternative land-use/transportation proposals.

The most basic traffic assignment algorithm is the single-pass, all-or-nothing assignment. The all-or-nothing algorithm assumes that the link impedance for each link of the network is fixed and that the impedance (travel time) is not affected by the magnitude of link flow nor is link flow constrained by link capacity. All the trips of a specified O-D pair are assigned to the links comprising the path which yields the minimum total impedance. As shown in Figure 1, the algorithm does not readjust link impedances. The procedure is adequate for the evaluation of major transportation/land-use alternatives, for the planning analysis of a transportation network given a projected travel demand, and for preliminary evaluation of a proposed land-use pattern given a transportation network.

A major weakness of the all-or-nothing procedure is the assumption of unlimited link capacity. The procedure may result in over-assigning volumes to a number of links and under-assigning volumes to other links in a network. Another weakness is that it does not reflect the fact that there may be one or more other paths between a given origin and destination which have a total impedance only slightly less than the minimum path. The

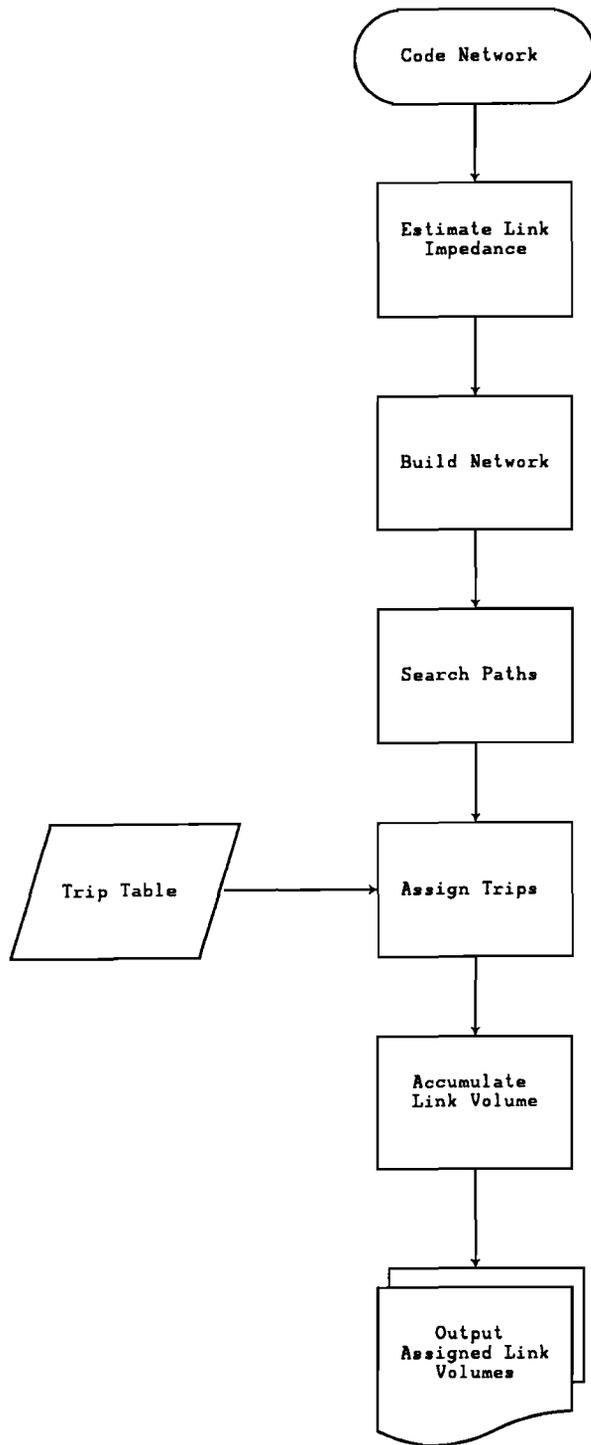


FIGURE 1. Basic Single-Pass, All-or-Nothing Assignment Procedure.

capacity-restraint procedure is more realistic in that it attempts to reflect the presence of traffic congestion and its effect on traffic patterns. Although there are variations in the capacity-restraint procedure, they can be simply regarded as a reiteration of the single-pass, all-or-nothing assignment. Based on their mechanism differences, the capacity-restraint assignments are generally classified in two categories: iterative or incremental. As shown in Figure 2 and Figure 3, both methods repeat the single-pass according to the associated link impedance functions after the link impedance functions are updated.

The equilibrium assignment procedure is a variation of the capacity-restraint procedure using successive all-or-nothing assignments which are weighted so as to attempt to achieve a stated objective function. In 1952, Wardrop proposed two criteria based on journey times that can be used to determine the distribution on travel routes (1):

1. The journey times on all the routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route.
2. The average journey time is minimum.

The first criterion is known as the principle of equal travel times. This principle implies that traffic will tend to move toward an equilibrium situation in which no driver can reduce his or her journey time by choosing a new route. The second criterion is equivalent to the principle of minimizing total travel times. A network system is considered the most efficient when the second criterion is satisfied, i.e., the total vehicle-hours are minimized.

These two principles may sound attractive from a theoretical point of view. However, it is well recognized that travel time in the strictest sense is not the only motivation for drivers' choice of route. The motivation as to choice of route is often a matter of safety, driver comfort, or a variety of other considerations and not just the minimum travel time. Consequently, it is extremely doubtful if the final pattern of traffic in a road network conforms to Wardrop's principles. In fact, there is no reason to believe that a "final" static equilibrium traffic pattern for any roadway system exists.

Multipath (stochastic) assignment techniques were developed in recognition of one or more of the following:

1. The impedance (travel time) on a link, or series of links, is a random variable and not a fixed value.

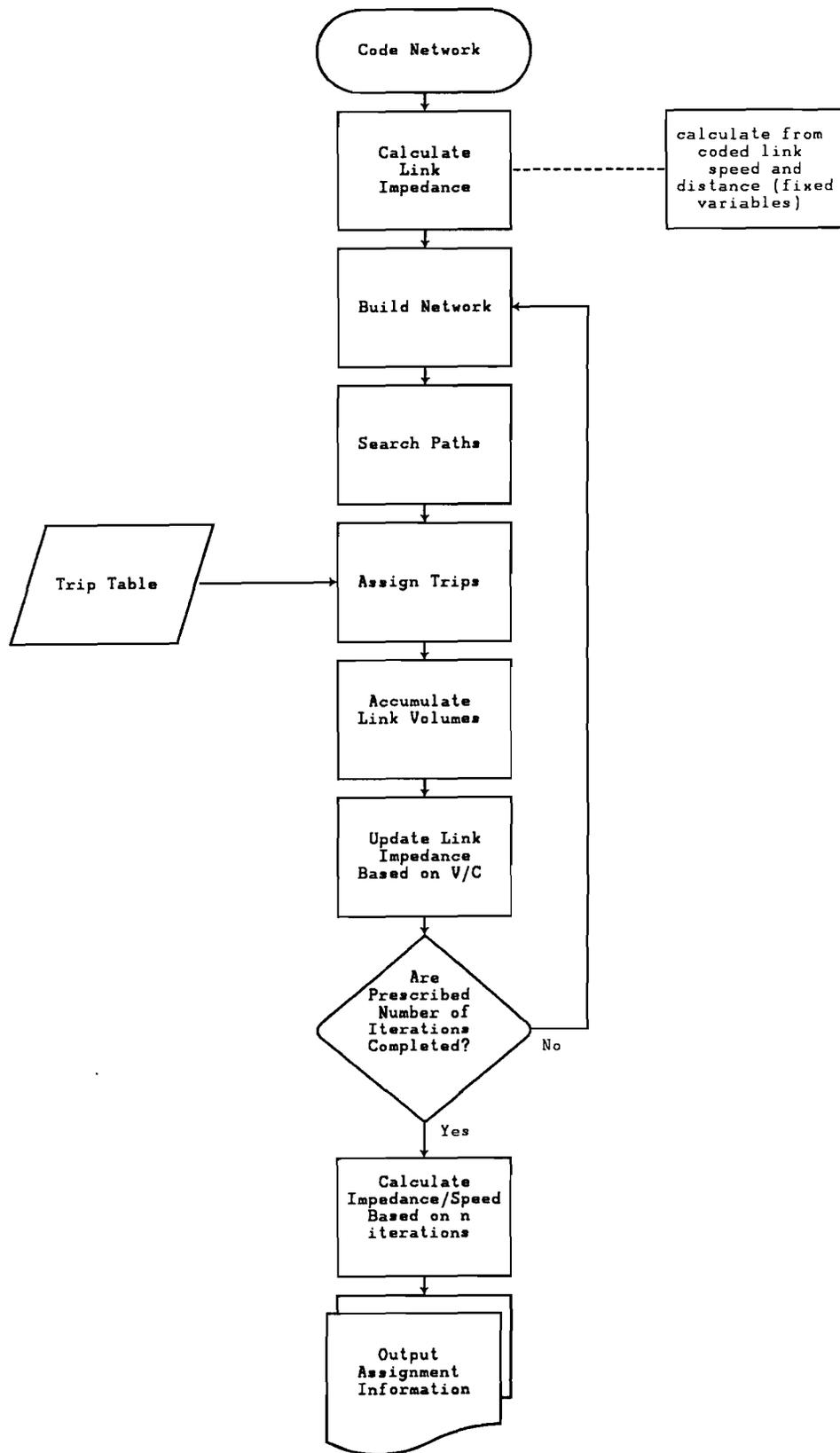


FIGURE 2. General Iterative Capacity-Restraint Procedure.

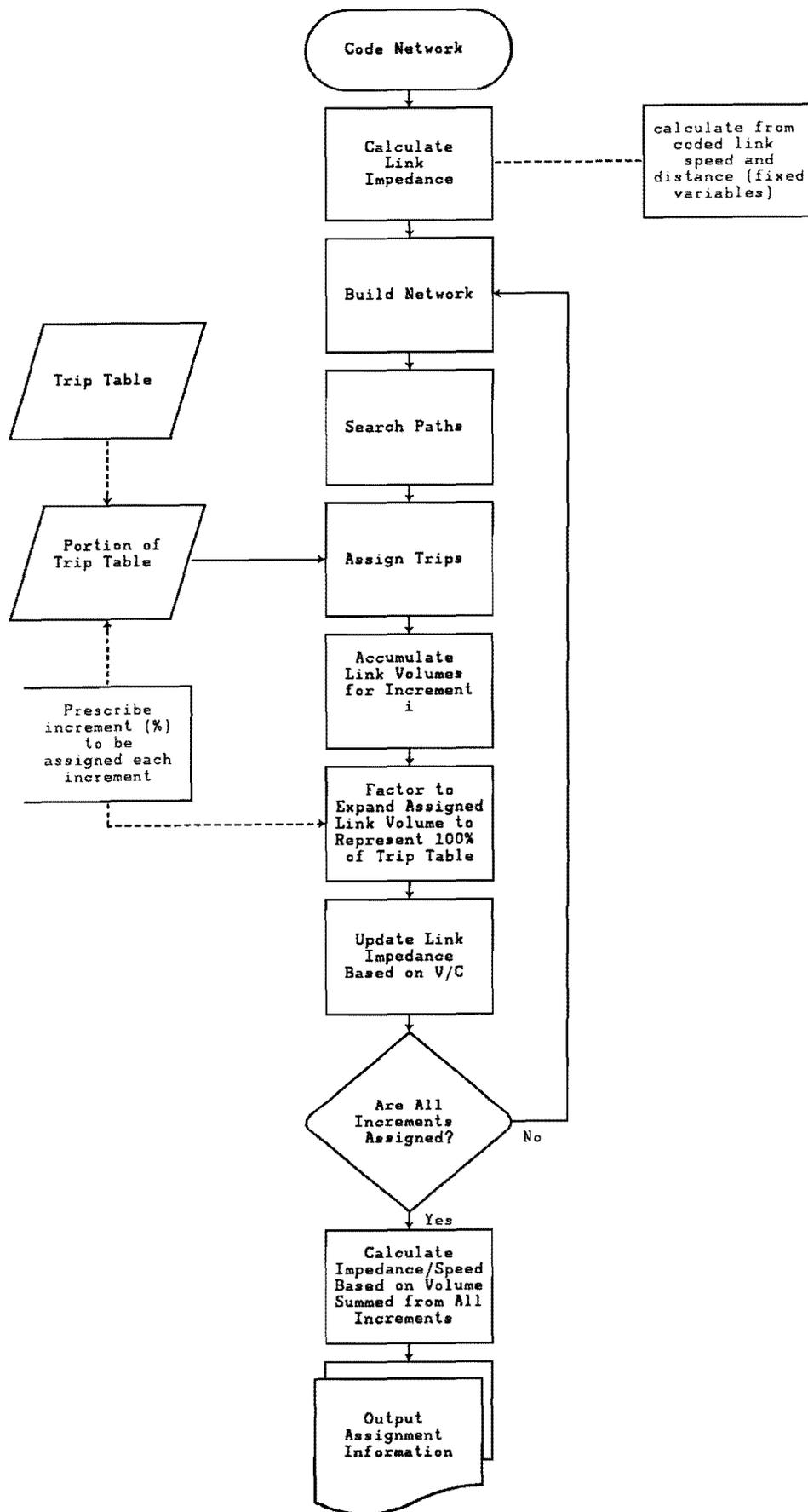


FIGURE 3. General Incremental Capacity-Restraint Procedure.

2. For most zone pairs, there are a variety of paths for which the travel times are similar.
3. Drivers do not, and cannot, differentiate between paths for which the travel time (impedance) differs by a small amount.
4. Where drivers have a multiplicity of choices, the specific route taken will vary depending upon conditions encountered on a given trip.

Most multipath assignment techniques are based on one of two approaches: either path enumeration or path diversion. The path enumeration model primarily reiterates the assignment procedure with variable link impedance inputs. Burrell's algorithm (2) is a typical path enumeration model in which the link impedances are assumed to be randomly distributed to account for errors in the driver's perception in link travel time. The path diversion model assigns trips to alternate paths without repeating the assignment procedure. The most noted path diversion model is Dial's algorithm. Dial's technique (3) originated from logit discrete choice theory in that each "reasonable" path between a particular O-D pair is assigned a portion of the O-D trips according to a route-choice probability.

In reality, any transportation network involves a dynamic process in which demand (traffic flow) and supply/performance (travel cost/time) is an interactive system. The all-or-nothing assignment with fixed link impedances is not capable of reflecting this interaction. Other assignment techniques such as the incremental, iterative, equilibrium, and path enumeration multipath assignments attempt to reproduce the interaction by repeating the capacity-restraint process. In theory, the iterative process continues until the adjusted link volumes and costs conform to, or as close as possible to, the presumed flow/cost relationship. In the literature, this is referred to as the convergence of the process. In practice, this would be a very lengthy computation, and this degree of refinement in the assigned volumes is not cost effective. The problem, then, is when does one consider the assignment process to be completed? Also, should convergence be achieved or is oscillation acceptable? Before these problems are examined further, two widely-applied capacity-restraint procedures, iterative and incremental, are discussed.

ITERATIVE CAPACITY-RESTRAINT PROCEDURE

The iterative capacity-restraint procedure repeats iterations of the all-or-nothing assignments in which all trips are assigned to the network on each assignment. Between each iteration, all link impedances are updated based on the last assigned link volumes and the prescribed link impedance function(s); the updated link impedances are then used in the next iteration. The resulting network-assigned volumes for all iterations are averaged to determine the final flow volumes on each link.

In short, the iterative assignment technique involves a number of successive adjustments (build network, search minimum paths, load network, and impedance) to obtain the assigned link volumes on a coded transportation network. As shown in Figure 2, the iterative capacity-restraint procedure consists of the following steps:

1. Calculate the initial link impedances from coded link speeds and distances.
2. Build minimum path trees based on the current link impedances.
3. Assign all trips (the entire O-D table) to their designated minimum paths.
4. Accumulate the assigned link volumes for each link.
5. Update link impedances based on the presumed link impedance functions in the form of volume-to-capacity ratio (V/C).
6. Repeat Steps 1 through 5 until the prescribed number of iterations is completed.

The adjustment of link impedances to reflect operating conditions (congestion effects) usually uses the latest assigned link volumes. In some cases a weighted mean of the impedance on the last as well as previous iterations is used. There are various views as to the appropriate number of iterations to best approximate the actual traffic conditions; however, four iterations are generally considered to be sufficient. Research by Humphrey (4) found that at least three to four iterations is desirable to apply capacity-restraint procedures, and reasonable assignment results are obtained by using an average of four loadings. Unpublished research by Stover et al., (5) of the Texas Transportation Institute produced similar conclusions.

INCREMENTAL CAPACITY-RESTRAINT PROCEDURE

In practice there are two types of incremental capacity-restraint procedures. In the first procedure, an individual tree is built for a randomly-selected centroid, and trips from this centroid to all other centroids are loaded. The base travel times on the individual links are then adjusted according to the link capacity function. Then, a tree is built for another randomly selected centroid, and the corresponding trips are assigned. This procedure, sometimes referred to as a one-pass incremental process, was first developed by Schneider for the Chicago Area Transportation Study (6). In practice it has been found that one tree loading has very little impact on impedance adjustments. The actual procedure used in most cases is that the travel times on the individual links are adjusted after trips from a number of centroids have been loaded.

A disadvantage of this type incremental assignment is that the variability in results depends upon the order in which centroids are selected. Thus, for the same O-D table input, diverse final assignment results can be obtained from consecutive runs of the model. In other words, the model may lack "robustness." In addition, once an increment of flow between a given centroid pair is assigned, it cannot be re-assigned to another path.

In the second type of procedure, increments (fractions) of the total O-D table are loaded successively to the minimum path trees built for all centroids. As shown in Figure 3, a portion of the O-D table is first assigned to the minimum path trees for all origin centroids using the initial coded travel cost (travel time). The assigned volumes are then factored to present a 100 percent loading, and the link impedances are adjusted according to the link capacity functions. A new network is then built; new minimum paths are built using the updated link impedances; and another increment of the trip table is loaded. The process is repeated until the entire trip table has been exhausted.

This method overcomes the disadvantages of the first type of incremental assignment and is used in most traffic assignment packages. The user has the option of specifying the percentage of trips to be loaded on each increment. In practice, faster convergence is obtained when decreasing fractions of the total demand are assigned with each successive increment.

THE PROBLEM OF CLOSURE (OSCILLATION)

As previously mentioned, capacity restraint is the process of changing the link impedances to reflect the interaction between traffic demand (flow) and traffic performance (impedance). In theory the iterative process continues until the adjusted link volumes and link impedances mutually conform to, or as close as possible to, some presumed flow/impedance relationship for each link. This is usually referred to as the convergence of the process. In practice, however, convergence is difficult to obtain; under certain conditions, it is not even logical.

Various convergence criteria are adopted according to the analyst's objective. The strictest convergent solution for capacity restraint is defined as one in which a set of link costs C_a^* is found such that $C_a^* = C_a(V_a^*)$; that is, the costs that enter the assignment are the same as those that emerge. It implies that the solution points for the network will be exactly located on the flow/cost curve presumed. Proponents of this criterion argued that the objective of the link cost (or speed) change is to find just such a solution. If found, the process is sound; if not, the model is bound to have an instability problem (7). In practice, it appears that the convergent solutions defined above are very difficult to achieve. A more realistic criteria is to examine the magnitude of the change (percent difference) of the volumes or travel costs on links, or a combination of both, between successive iterations. The most widely used criterion in practice is simply a prescribed number of iterations or increments, usually three or four although as many as six or seven are sometimes used.

Another problem associated the convergence of the process is the so called "see-saw" pattern (oscillation). This may occur when a link has an initial low impedance and is assigned a relatively high volume; this in turn leads to a high impedance on the next iteration, and correspondingly, a low assigned volume. The "see-sawing" of volumes and impedances sometimes continues from one iteration to the next. Van Vliet and Dow (7) stated that the following problems are associated with oscillation:

1. The "best" point at which to stop the sequence is far from obvious. In calibration exercises one may, perhaps fortuitously, hit upon a solution which gives a generally good fit after, say, seven iterations; whether the same

procedure repeated in a forecast will lead to equally good results is open to question.

2. Oscillating loadings may cause expensive computer running time and cost.
3. Operational evaluation is made difficult.

The oscillation pattern, however, is common for a demand-performance interactive iterative process. Capacity-restraint procedures in which the impedances are adjusted individually based upon the assigned V/C ratio of the specific link can be expected to result in oscillation. In fact, "closure" is not necessarily expected or even desirable. The pattern of assigned link volume and link speed provide the analyst with valuable information for evaluating the compatibility between the projected land-use activity and the transportation network.

There are three patterns in the change in link volumes and impedances (or speeds) that result with capacity-restraint assignment:

1. Oscillation occurs from iteration to iteration. This results when the network or a portion of the network is approaching congestion, and there are alternative routes between groups of centroids. For example, if a series of links are under-assigned, the BPR impedance adjustment will reduce the impedances; the links are over-assigned on the next iteration. Then, the impedances of these links are increased on the next network update; and the links are, consequently, under-assigned. This oscillation indicates that the network has adequate capacity.
2. A route is over-assigned on successive iterations even though the link impedances are continuously increased on successive iterations. This indicates that the projected land use generates more traffic than can be accommodated by the proposed network or that there are no alternative routes to serve the corridor. This pattern demonstrates that the proposed land-use activity and the proposed network are not compatible and some other land-use/transportation alternative needs to be considered.
3. A route is continuously under-assigned on successive iterations. The assigned link volumes do not increase, or increase very little, although the link

impedances are continually decreased on successive iterations. This indicates that for the projected land-use activity, there are very few trips which can utilize the route. This condition is commonly observed in the fringe of the study area. The pattern of continual under-assignment can result in very low link impedances and, therefore, imply unrealistically high speeds. It can be avoided by using an impedance adjustment that increases the link impedances when links are over-assigned but does not change the impedances on links which are under-assigned. For example, the impedances for the under-assigned links are calculated from the coded speeds.

In summary, capacity restraint is the process of changing the link impedances to reflect the interaction between traffic demand (flow) and traffic performance (impedance). In theory the iterative process continues until the adjusted link volumes and link impedances mutually conform to, or as close as possible to, some presumed flow/impedance relationship for each link. In practice this convergence is very hard to obtain; and, under some conditions, it is not even desirable. However, various path algorithms other than the minimum path, such as Dial's or Burrell's algorithm, can be incorporated into the capacity-restraint iterative process. In this way the iterative cycle involves repeating the sequence of multipath routing and link volumes loading.

III. MULTIPATH PATH PROCEDURE

EXISTING MULTIPLE PATH METHODS

In general, most multipath assignments producing multiple paths or similar effects are based on one of two approaches: path enumeration or path diversion. Path enumeration models primarily reiterate the assignment procedure with different link impedance inputs. Path diversion models divert trips to alternate paths without repeating the assignment procedure.

As shown in Figure 4, the single-pass assignment procedure includes seven basic steps: 1) code network, 2) estimate link impedance, 3) build network, 4) search paths, 5) assign trips, 6) accumulate link volume, and 7) output assigned link volumes. Variations can be initiated at either link impedance estimation (Step 2) or path searching (Step 4) in order to produce multiple paths or similar effects. Path enumeration models are developed by making the variation at Step 2; the multiple paths are, in fact, obtained from different networks built from successive assignment processes by changing the link impedance estimation iteratively. As shown in Figure 5, Burrell's single-pass algorithm generates multiple paths from successive estimations of the link impedances.

On the other hand, the multiple paths of the path diversion models are generated by multipath searching algorithms, such as Dial's algorithm (3) or Gunnarsson's algorithm (8), in which the single-pass assignment process is not repeated. Thus, the multiple paths are produced "simultaneously." The conceptual flow diagram of Dial's algorithm shown in Figure 6 and Gunnarsson's shown in Figure 7 indicate that the multiple paths of these models are produced by their path searching mechanisms.

The general capacity-restraint assignment, which builds different networks by updating link impedances based on V/C ratios at each iteration, can be regarded as the most popular path enumeration model. It uses the minimum path algorithm to load a network and, in turn, the results of the loading to update the link impedances. The updated link impedances are then used for a subsequent all-or-nothing loading in which new "shortest" paths are to be found from the newly built network. The multiple paths are implicitly obtained from these successive heuristic processes. Applications of this method have been widely implemented since the late 1950's (6,9,10,11,12,13).

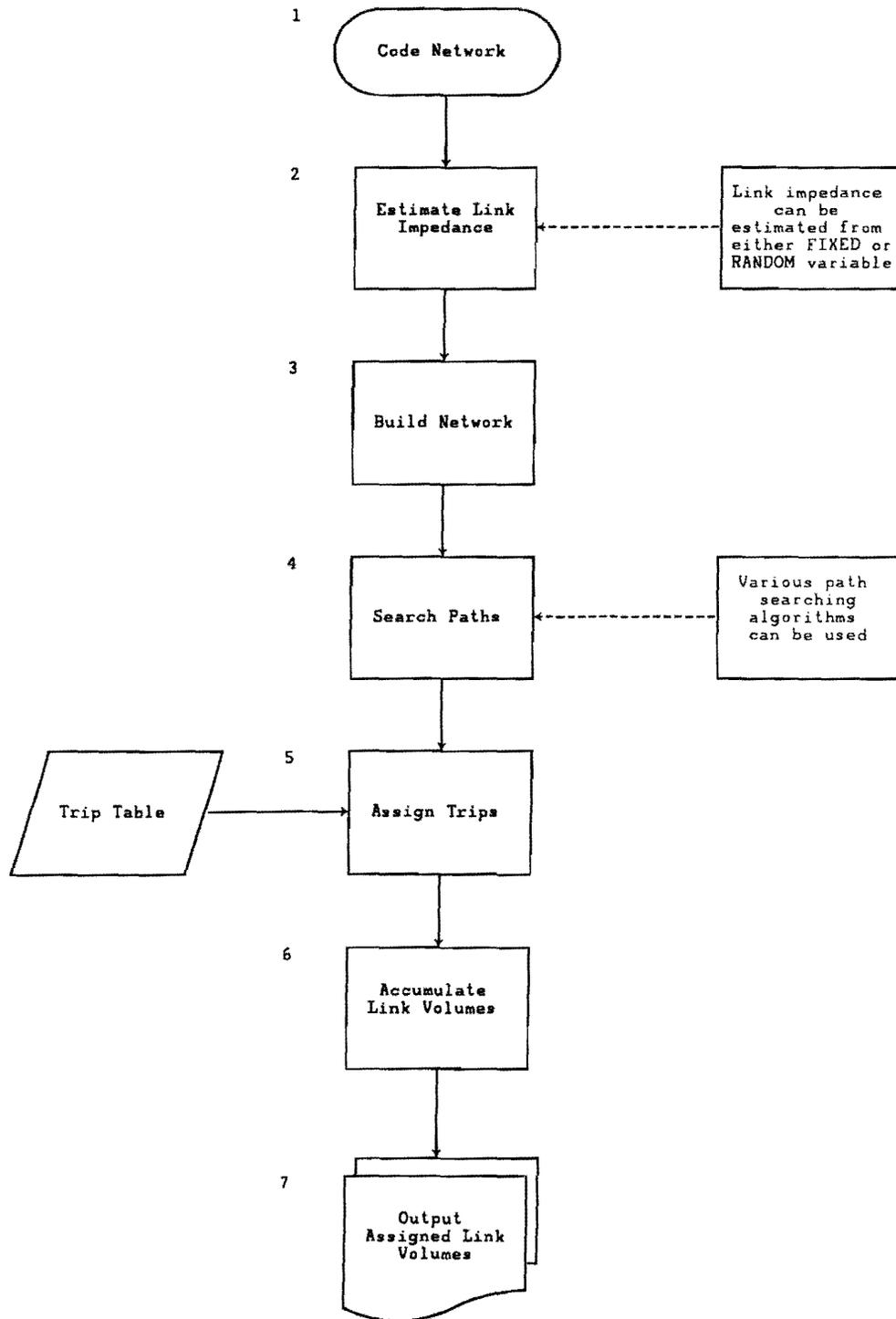


FIGURE 4. General Single-Pass Assignment Procedure.

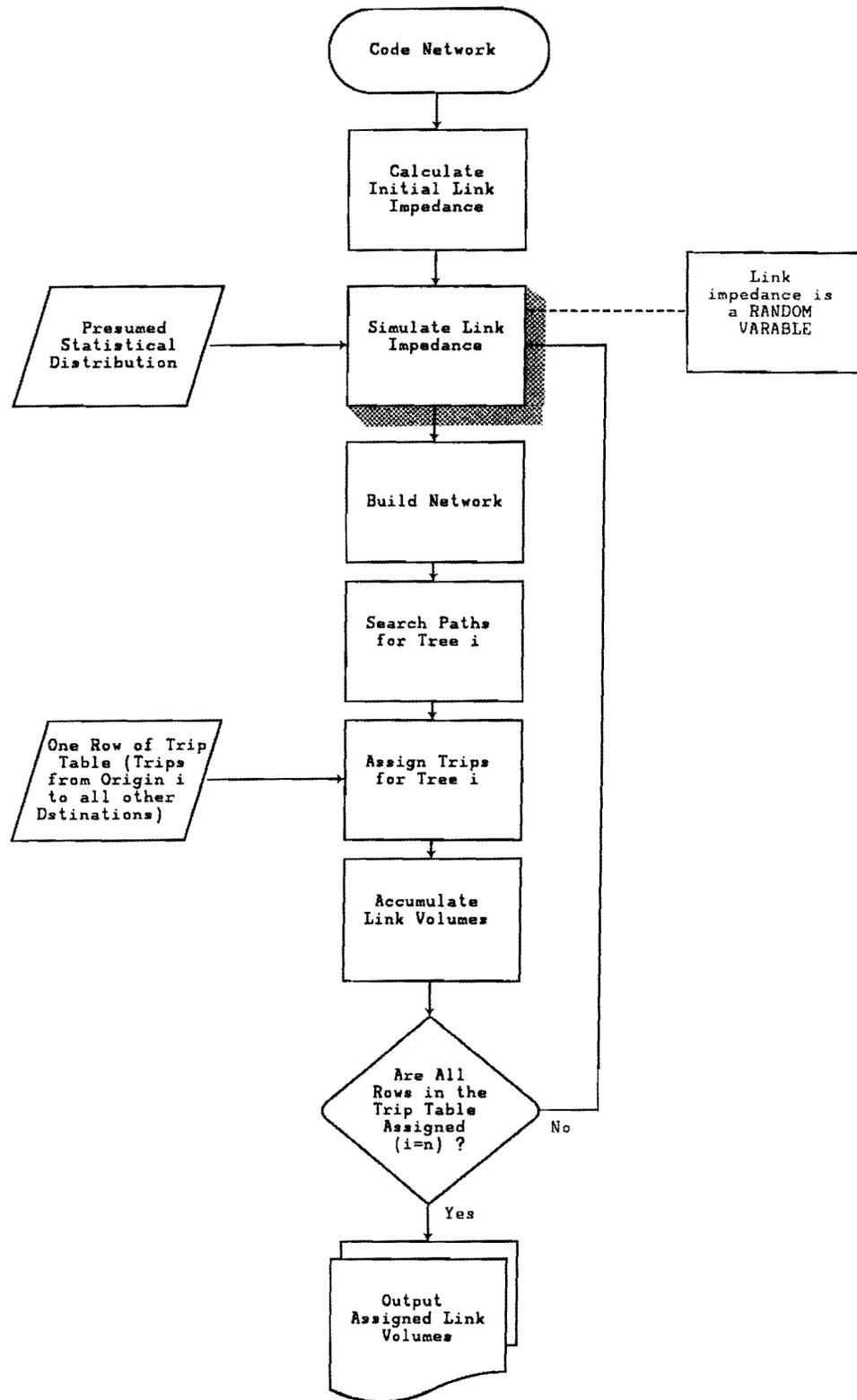


FIGURE 5. Burrell's Single-Pass Assignment Procedure.

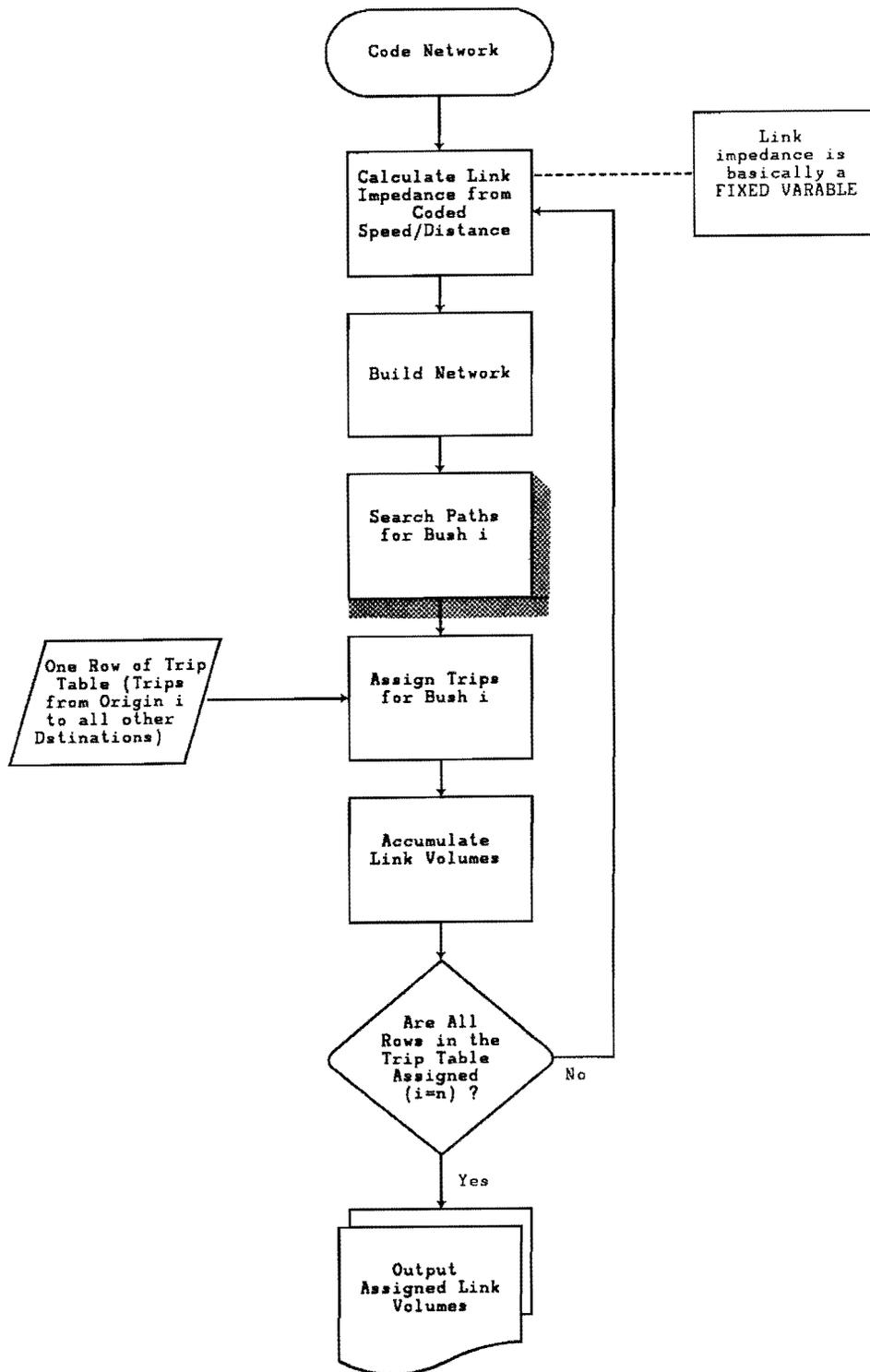


FIGURE 6. Conceptual Diagram of Dial's Algorithm.

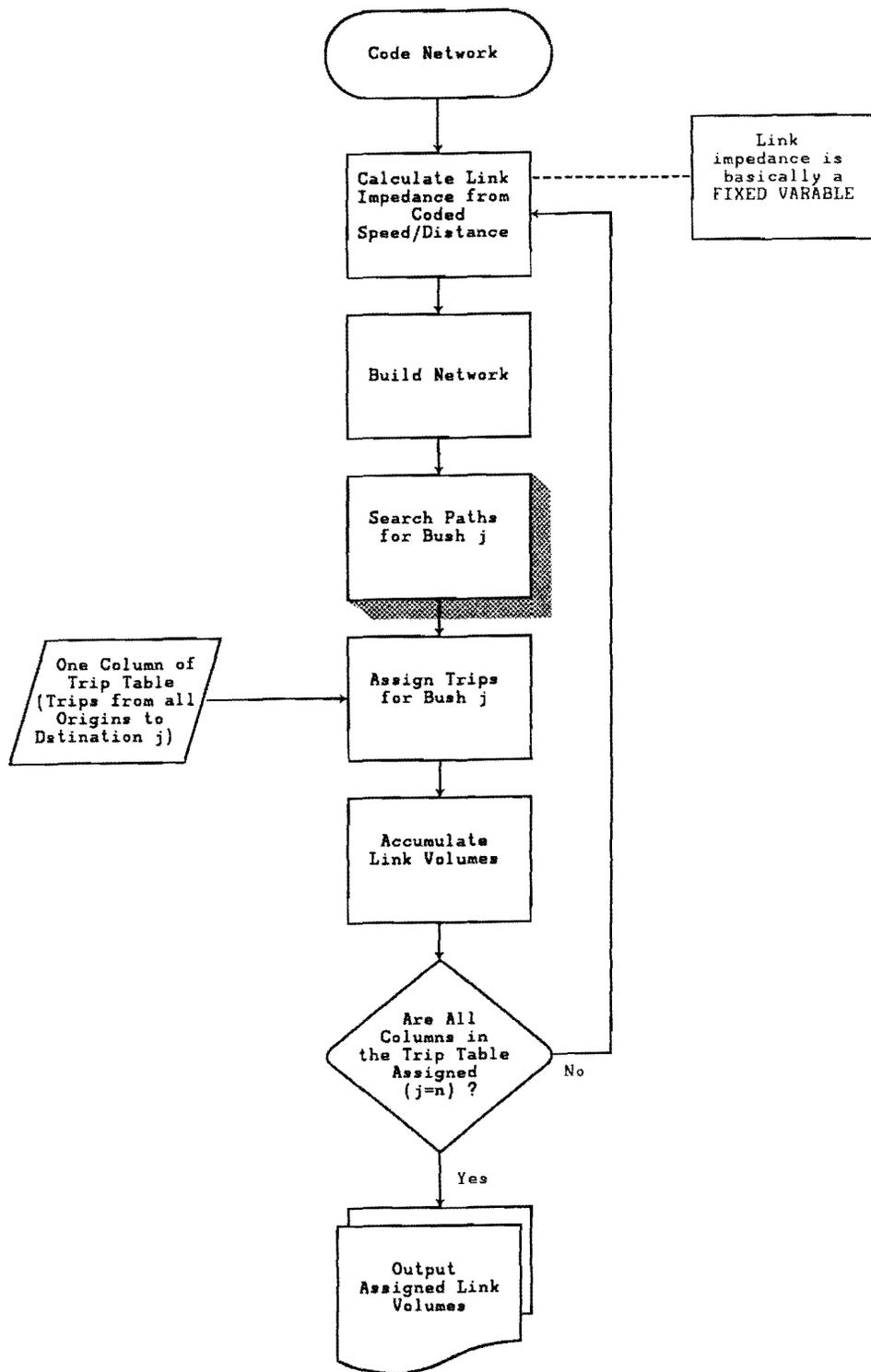


FIGURE 7. Conceptual Diagram of Gunnarsson's Algorithm.

One major criticism of the method is that it does not take certain unpredictable factors of drivers' behavior into account. Instead of searching different paths for each O-D pair using a multiple path algorithm, the method generates only one "shortest" path for each centroid pair at each iteration. An alternate path for any specific centroid pair is acquired only when it can yield less link impedance than the shortest path of the previous iteration. In reality, trip makers do not choose routes in this fashion when there are alternate routes having similar travel times between an origin and a destination. Some traffic will use each route. Even utility-maximizing drivers sometimes also use alternate paths due to the deficiency of perfect information of roadway conditions and estimation errors.

In order to encompass these uncertain factors of route selection, Burrell proposed a multiple path assignment method which affiliates random errors with the link impedance estimation (2). The method assumes that the driver does not know the actual link travel times, but that the driver perceives a "supposed time" for each link. The supposed link time is drawn at random from a statistical distribution with the observed mean travel time and standard error. Burrell's method can be regarded as an advancement of general capacity-restraint methods in that it presumably generates more alternate paths with the variations of link impedance. Several applications of the method have been carried out, mostly in England and Canada (14,15,16,17). Similar efforts have been tried in the United States (18,19).

The path diversion models divert trips to alternate paths without repeating the assignment procedure. The concept of "diverting" traffic has primarily evolved from the "diversion curve" developed in the 1950's (20,21,22). The diversion curves were used to forecast the usage of a proposed freeway (compared with an existing arterial) in such a way that the probability of freeway usage is related to time saved and other measures of disutility. This methodology was suitable for the primary "two-route" analysis stage; however, it is inadequate for network analysis where several alternative routes may be available.

Dial's algorithm and Gunnarsson's algorithm are the most renowned path diversion models applicable in network system analysis. Dial's algorithm is interpreted as a probabilistic choice model that proportionately assigns link volumes to "reasonable" routes.

Gunnarsson's algorithm is based on a similar concept but a different approach which will be explained later. The mechanism of both algorithms is based on the principle of the Markov chain which implies a stochastic process. Using the Markov chain, the future is determined from the present state and is independent of the past. Therefore, the path choice probability can be transferred from node to node. The estimation of choice probability is calculated by using an exponential (logit) type formula in Dial's algorithm, whereas it is expressed in terms of an attractivity (link resistance) model in Gunnarsson's algorithm.

A major criticism of the Dial's probabilistic assignment model is that it assumes that alternate routes in the route choice set are independent. This assumption profoundly conflicts with reality in that network links are usually correlated, and routes unavoidably overlap each other. It is speculated that the same problem would occur in Gunnarsson's algorithm since it is based on a similar concept. Because they are the major exemplary multipath algorithms developed and applied, more details of Burrell's, Dial's, and Gunnarsson's algorithms will be discussed in the following sections

BURRELL'S ALGORITHM

The algorithm developed by Burrell reiterates the assignment procedure with variable link impedances. Therefore, the algorithm is classified as a path enumeration model in this report. Burrell makes the following assumption relative to drivers' knowledge of travel time:

It is assumed in the multiple route model discussed here that the user does not know the actual link times, but that he associates with every link in the system a supposed link time, which is drawn at random from a distribution of times, having as its mean the actual link time, and a mean deviation, which might be say 20% of the actual link time. (2)

In Burrell's procedure, the supposed link travel time is drawn at random from a distribution of times having as its mean the measured or estimated average link time and a mean deviation which can be stated as some percentage of the average travel time for the link. The spread of travel time for each link is chosen so that the ratio of mean absolute deviation to actual link time is the same for all links in any particular category. This ratio

is referred to as the diversion factor. Thus, based upon Burrell's description it appears that the link cost distribution applied to a link of cost C may have a number of variations of the form:

$$C^* = C (1+y_i*D) \tag{3.1}$$

where:

C^*	=	the random value of link travel cost
C	=	the actual link travel cost (used as the mean observed cost)
y_i	=	a probability distribution with zero mean and unit mean deviation
D	=	the diversion factor

In his paper, Burrell stated that for reasons of computation the method used eight possible values as a substitute for the probability distribution y_i . With only eight values, it is not possible to adequately represent the tails of a normal distribution. Thus, the author used a simple rectangular distribution with each of the eight possible values having an equal chance of being chosen (2).

However, it is unrealistic to apply the same diversion factor to all the links in a network regardless of their lengths. For example, on a journey for which the best possible time is five minutes, a person might choose a route taking seven minutes which is 40 percent greater than the best time. On the other hand, it seems much less likely that on a journey for which the best time is 100 minutes, a person would choose a route taking 140 minutes. Burrell suggested that the total of supposed link times for long routes will, on average, differ from the total of actual times by a smaller percentage than for shorter routes.

Later applications of Burrell's procedure proposed an alternative to account for the route (link) length problem (16). The values specified in the distributions are proportionate deviations from the actual link times (the mean of the observed link travel times) specified in the network. In this way, the degree of deviation from the coded cost applied to a section of road is independent of the number of links into which it is divided. Each link obtains its random values by varying the mean deviation according to the link costs (times) and its class. Mason (16) describes the form of the link cost spread in terms of the square root of the actual link cost:

$$C^* = C \left(1 + y_i * \frac{k}{\sqrt{C}}\right) \quad (3.2)$$

where: C^* = the random value of link cost
 C = the actual link cost (the mean observed link cost)
 y_i = a probability distribution with zero mean and unit mean deviation
 k = the mean deviation applied to a link of unit cost

In theory, the user controls the amount of multiple routing by specifying k/\sqrt{C} which is the ratio of the mean deviation of each link cost distribution to the coded link cost. Considering that each path consists of several links, it is very unlikely that the total travel time for any path will have an extremely low or high travel time (impedance or cost).

According to Burrell, the multiple route procedure can be used for either restrained or unrestrained capacity situations. In uncongested networks, there is no need for any iteration of the single-pass assignment process since the volume of traffic in the network is not sufficiently great to affect its speed. Burrell's single-pass assignment procedure, as previously shown in Figure 5, repeats the randomization of link impedances for each origin zone. For each origin zone i in turn, link impedances are randomized for all links in the network based on the presumed link impedance distribution(s). The minimum-path tree for the zone is built, and traffic from the origin centroid to all other centroids is loaded. Then, link impedances are again randomized for all links, and the minimum path tree is built for another origin zone. The procedure is repeated until traffic from all origin zones has been loaded. In short, Burrell's single-pass procedure generates a different set of link times for each origin zone.

Burrell states that the most variable application of the multiple route method is in capacity-restraint procedures. However, he does not present any further evidence to support this statement. Improvement in the assignment results would presumably result from more logical minimum paths than the conventional capacity-restraint procedure since it applies additional link impedance variations (randomizations) after link impedances have been updated at each iteration. Figure 8 shows the incorporation of Burrell's algorithm into the

conventional iterative capacity-restraint procedure. Unlike the conventional iterative restraint procedure, the link impedances used to build networks are a random variable simulated from the assumed statistical distribution. In addition, multiple paths between nodes are generated due to an inherent property which Burrell calls "link perturbation" of the algorithm.

In either restrained or unrestrained cases, Burrell's procedure simulates the random variation in drivers' estimates of cost by repeated randomization of link costs, each randomization being followed by the generation of a set of minimum paths. The technique used in building minimum path trees (or vines) is the same as that used for minimum-path (all-or-nothing) assignment. However, in Burrell's path searching process, multiple paths are presumed to be perturbed since link costs are randomized instead of fixed. Burrell used illustrations of four- and five-node simple networks to demonstrate that the link perturbation method would generate more "reasonable" routes compared to Dial's method (23).

The mechanism of link perturbation can be explained by a graphic example. Figure 9 illustrates a number of possible routes between nodes A and F. The mean deviation of each link is assumed to be proportional to the square root of its impedance. Consider the two routes ABCF and ABDCF. The latter route would never be selected because BDC would have a total time of 20 units which is well outside the possible value limit when compared with BC. In other words, the minimum possible value of BDC ($20 - \sqrt{20} = 15.5$ units) is still greater than the maximum possible value of BC ($10 + \sqrt{10} = 13$ units).

On the other hand, route AEF, which is a longer route than ABDCF, may well be selected when compared with route ABCF. The corresponding deviations of link costs show that the minimum possible value of AEF ($2 * (115 - \sqrt{115}) = 208$ units) is smaller than the maximum possible value of ABCF ($2 * (100 + \sqrt{100}) + (10 + \sqrt{10}) = 223$ units). This is considered realistic because it often occurs in an actual situation. Although several routes between A and F can be identified, only one route is selected for each pass through the tree-build process.

According to Burrell, a fundamental feature of the link perturbation method is that it generates multiple routes between node pairs. In the traditional minimum-path assignment, the path between any two nodes is the same for every tree including the node

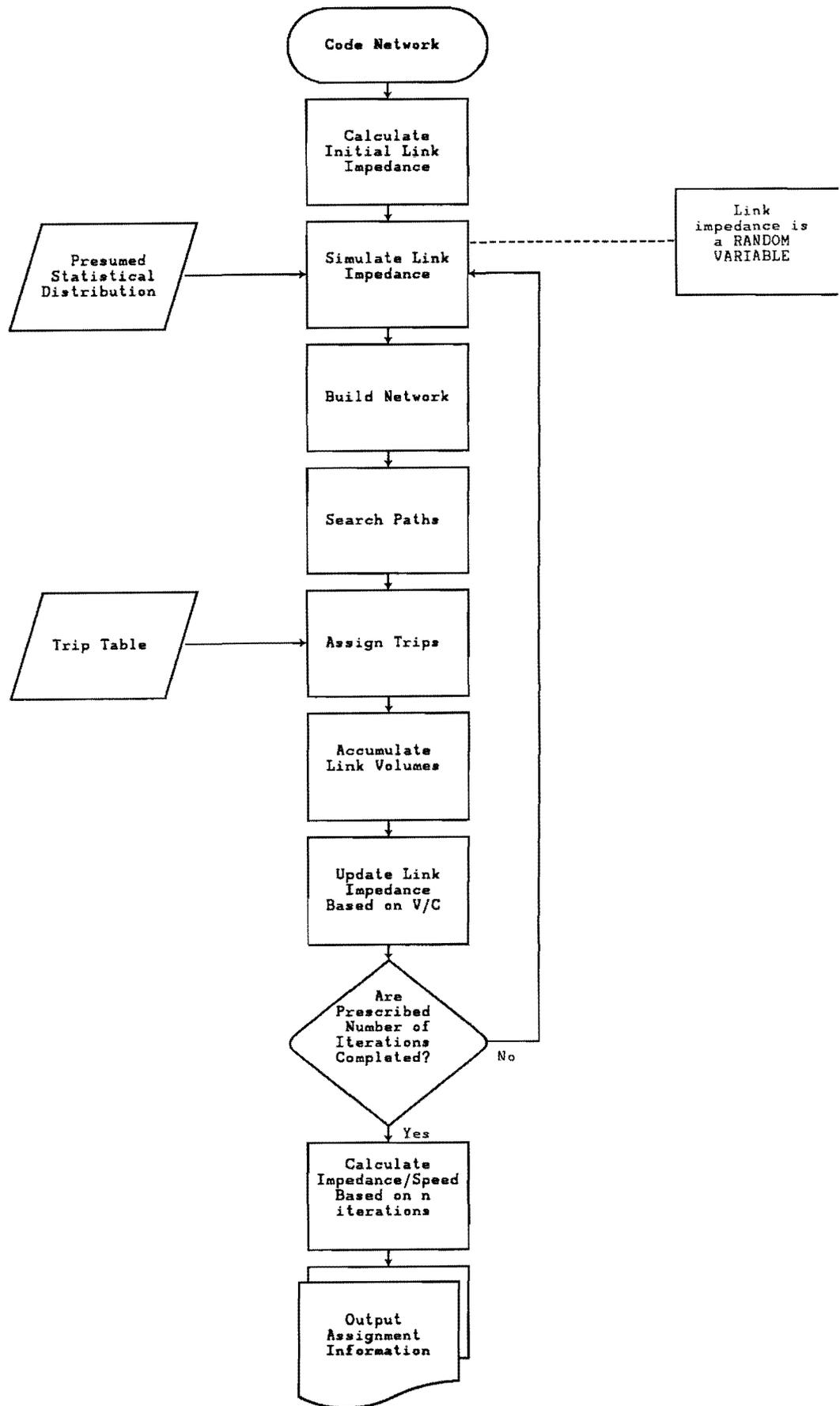
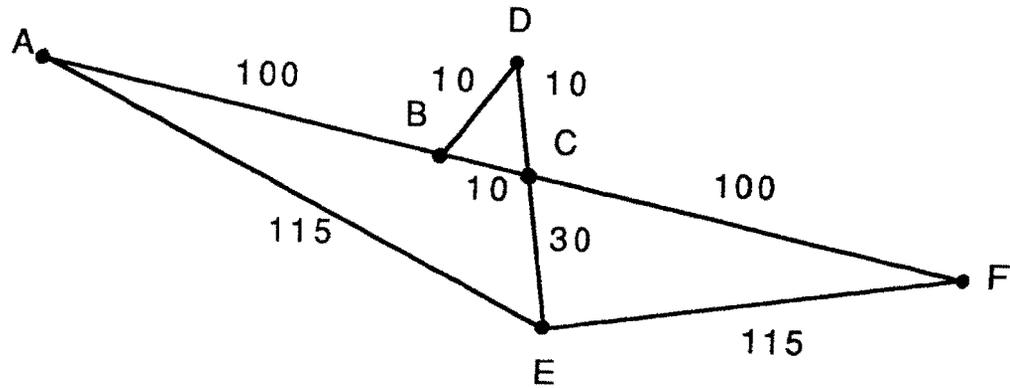
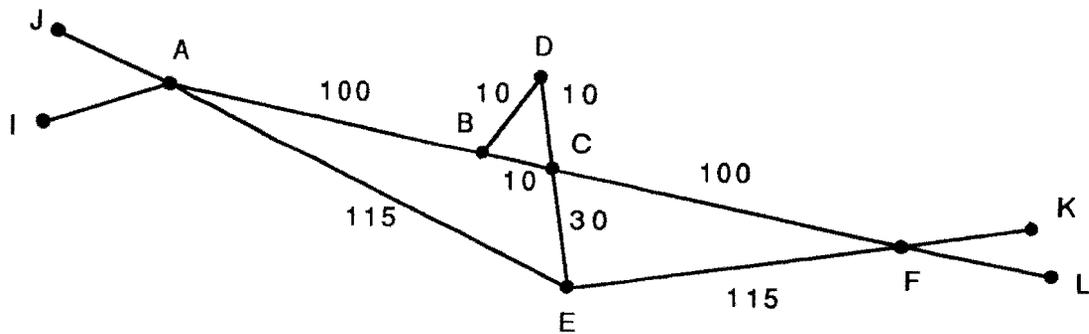


FIGURE 8. Burrell's Iterative Capacity-Restraint Procedure.



Source: Adapted from Burrell (2) and Dalton and Harmelink (14).

FIGURE 9. Example of Link Perturbations between Nodes A and F.



Source: Adapted from Burrell (2) and Dalton and Harmelink (14).

FIGURE 10. Example of Link Perturbations between Origin Zones and Destination Zones.

pair. In the link perturbation method, a different value may be chosen to build each successive tree; thus, several different paths may be found between any two nodes. As shown in Figure 10, trees are built from origin zones I or J to destination zones K and L. By using the minimum path method, the route selected between A and F will be the same (route ABCF) for both O-D pairs I-K and J-L; whereas, by the link perturbation method, the selected routes may be different depending on the randomized values of link costs between A and F for various origin-destination pairs. In this case, the selected route from I to K may be IABCFK or IAEFK. By the same token, the selected route from J to L may be JABCFI or JAEFL. As such, there is no longer a unique path between A and F.

Discussion

Burrell views link impedance in terms of the individual driver's knowledge as choice. An alternative interpretation of link impedance as a random variable is that it cannot be measured precisely. This is a more logical view since link travel time is subject to temporal variation. Moreover, measurement errors are inevitably associated with link travel time or other quantification of link impedance for any transportation network.

Burrell's algorithm started with an assumption of individual driver's behavior; it is assumed that the driver finds and uses the route which minimizes the sum of the supposed link times. However, Burrell states:

It is not possible in practice, for reasons of computation, to allow each user to have a different set of supposed link times, and thus [it] is necessary to have a group of users all having the same set of supposed link times. It is convenient to take as a group the trips generated in a particular zone. (2)

In theory, more satisfactory aggregate assignment results would be obtained if more than one tree were built for each origin zone. Later in Mason's (16) and Dalton and Harmelink's (14) application of the method, more than one tree is built per origin zone. However, in their applications, the impact of this variation on the overall assignment results as well as the cost-effectiveness of the additional tree(s) has never been evaluated.

The link perturbation mechanism appears very appealing in searching for node-to-node paths. However, it seems to have little impact on the overall assignment results. The

variations between individual trips tend to be balanced in the aggregation process. Burrell states:

The traffic assigned to a particular road link usually includes trips generated from many different zones. In a large network, the maximum proportion of trips on a link generated from any one zone is not often more than 10%, and is usually much less than this on important roads carrying long-distance traffic. Thus the effect of the random numbers used in obtaining the supposed link times for any particular zone of generation has only a small effect on the traffic assigned to any particular road link. (2)

Therefore, it is speculated that Burrell's procedure and the traditional capacity-restraint procedure are likely to produce different results at a particular iteration. However, there may be little difference between the overall assignment results of the two procedures.

In summary, Burrell assumes that 1) the user does not know the actual travel times on links but associates a supposed travel time; 2) the user finds and uses a route which minimizes the sum of the supposed link times; and 3) a group of trips originating from a particular zone have the same set of supposed link travel times and consequently there is only one tree for each origin zone. The procedure can be implemented with or without the capacity-restraint procedure. However, the assumption that randomized link costs are independent from link to link is in contradiction with the reality that the travel costs of successive links are, in fact, correlated with each other. Although the calibration of parameters is not as complicated as Dial's algorithm, the process requires observed traffic data from which the link cost distributions can be derived.

DIAL'S ALGORITHM

Dial's assignment algorithm has stimulated great interest among researchers and practitioners of transportation planning because it seems to offer immediate advantages over the all-or-nothing assignment algorithm. Instead of being concerned solely with minimum time paths in a network, the probabilistic multipath method deals with "reasonable" (or efficient) paths. The method assigns trips to all reasonable paths simultaneously in such a way that the resulting effect is identical to what would have been obtained had each path

been assigned trips separately under certain choice probability assumptions. The fraction of trips assigned to each alternate link is a diversion probability based on the comparative length and number of reasonable paths through the link.

The objective of Dial's algorithm is to obtain the link flows corresponding to the division of each set of O-D trips between a set of "reasonable" routes according to a logit-type formula. The formula defines the probability of choosing a route (P_R) as a share (proportion) of the whole choice set.

$$P_R = \frac{\exp(-\theta * \Delta_R)}{\sum_{R \in R_{O-D}} \exp(-\theta * \Delta_R)} \quad (3.3)$$

where:

P_R	=	the probability of loading a trip on route R
θ	=	a user-defined "division" parameter
Δ_R	=	the "excess" cost along route R compared with the minimum path cost
R_{O-D}	=	the choice set = all the "reasonable" routes from O to D

Five functional specifications are required for Dial's algorithm (3):

1. The model should give all reasonable paths between a given origin and destination a non-zero probability of use, while all unreasonable paths should be given a probability of zero.
2. All reasonable paths of equal length should have an equal probability of use.
3. When there are two or more reasonable paths of unequal length, the shorter should have the higher probability of use.
4. The model's user should have some control over the path diversion probabilities.
5. The assignment algorithm should not explicitly enumerate paths.

The first four of these requirements imply the existence of a working definition of a reasonable path. A path is assumed reasonable if it is "efficient." An efficient path is one which does not backtrack; that is, as it progresses from node to node, it always gets farther from the origin and closer to the destination.

In short, Dial restricts "reasonable" to those that continually move 1) further away

from their origin "O," and 2) closer to their destination "D." In his second algorithm, the second restriction is removed saving substantial computer running time (3). This relaxation allows the algorithm to search paths and load O-D trips at one execution for each row of the O-D table. In contrast, the first algorithm executes the whole procedure once for every cell of the O-D table. For instance, to load a 100 x 100 trip matrix, the first algorithm needs to execute 10,000 times; whereas the second algorithm needs to execute only 100 times.

The primary algorithm (either the first or the second algorithm) to obtain the corresponding link flows is divided into a "forward pass" and a "backward pass" for each origin O . As shown in Figure 11 (the flow chart of the second algorithm), the forward pass performs path searching and the backward pass executes trips loading in general. The forward pass includes the following procedures:

1. Initialize all nodal "weights" $W_A = 0$ except for the origin node where $W_O = 1$.
2. Consider each node B in the network in order of increasing cost from the origin. For each link AB entering node B set:

$$\left\{ \begin{array}{l} W_{AB} = W_A \exp(-\theta \Delta_{AB}) = W_A \exp[-\theta(C_A^* + C_{AB} - C_B^*)] \text{ if } C_A^* < C_B^*, \text{ or} \\ W_{AB} = 0 \text{ if } C_A^* > C_B^*; \end{array} \right.$$

$$W_B = \sum_A W_{AB}$$

where: C_A^* and C_B^* = the minimum cost from O to A and B , respectively
 C_{AB} = the cost on link AB
 Δ_{AB} = $C_A^* + C_{AB} - C_B^*$
 Δ_{AB} = the excess cost incurred by a trip going from O to B through A rather than through the minimum cost route

The forward pass terminates when the destination is reached.

The backward pass includes the following procedures:

1. Initialize all "cumulative" volumes $V_B = 0$ except for those nodes which are destinations, in which case $V_B = T_{O-D}$ = the total number of trips from O to D .

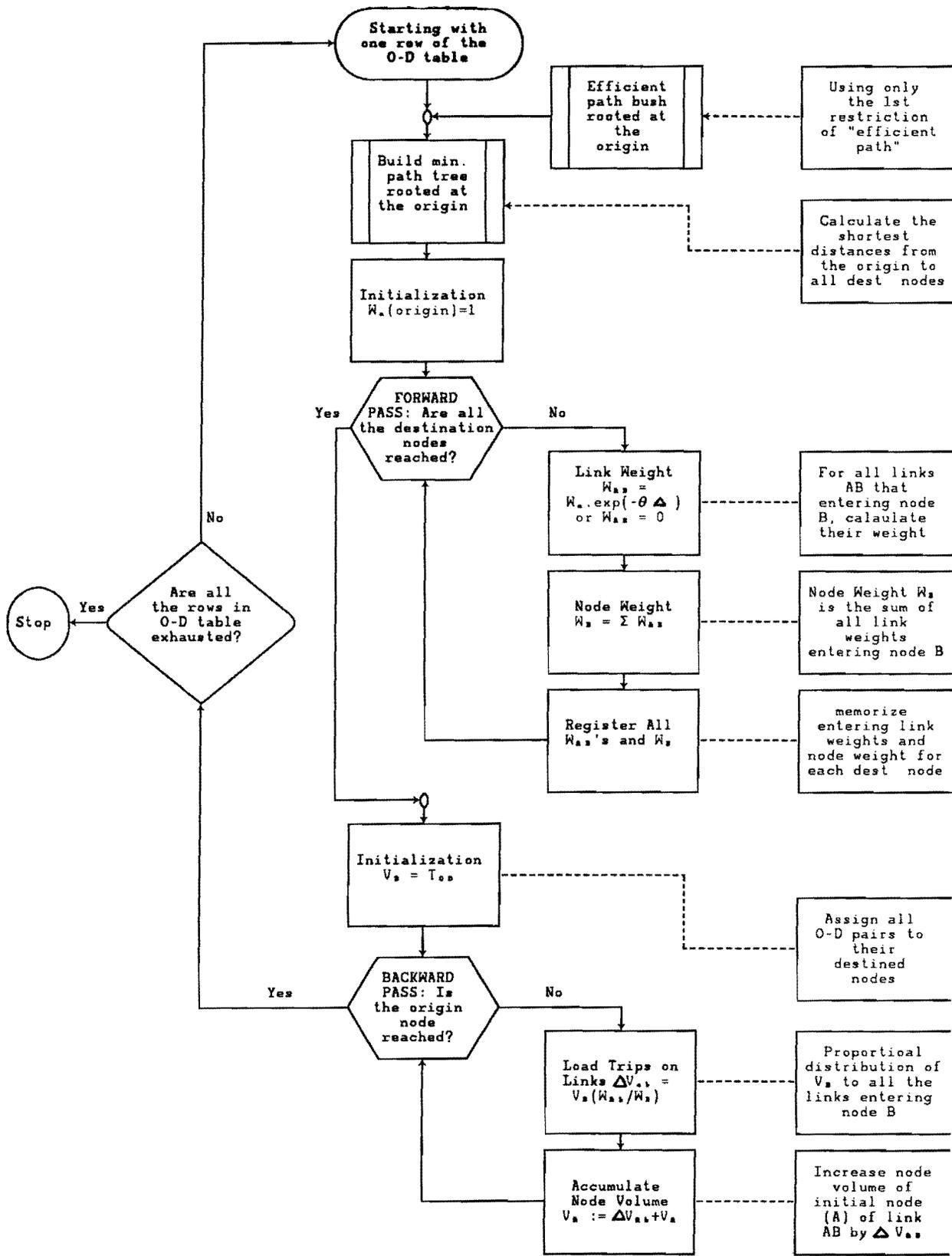


FIGURE 11. Flow Chart of Dial's Second Algorithm.

2. Consider each node B in order of decreasing cost from the origin, dividing the accumulated trips V_B according to:

$$\Delta V_{AB} = V_B * W_{AB} / W_B \text{ and incrementing } V_A \text{ by } \Delta V_{AB}.$$

The backward pass terminates when the origin O is reached.

Discussion

The "probability" of using a route (Equation 3.3) proposed by Dial is possibly derived from the logit-based discrete choice model (also known as the random utility choice model). Dial's algorithm appears to be a genuine probability. However, researchers argue that it is not a probability but rather a proportion (24). Simply, it can be shown that in the forward pass all the link travel times are pre-determined and fixed and, thus, produce flows that are deterministic.

This deficiency primarily stems from the underlying assumption of the logit-based model and is not unique to Dial's assignment model. In discrete choice models, different assumptions about the probabilistic distributions of the random components of the utility function will lead to different operational models. The logit model is derived from the assumption that the error terms in the utility function for different alternatives are identically distributed with the same fixed variances. It is not applicable in cases where the error terms' distribution depends on the characteristics of alternatives. Therefore, Dial's model cannot accommodate the characteristics of alternative routes. For instance, the model cannot account for the relationship of route travel times to the respective length of those routes.

According to Dial's exponential formula (Equation 3.3), the same probability of trips assigned to an alternative path (P_R) will be obtained for different choice sets as long as the same excess costs (Δ_R) are present. For example, the probability (proportion) of a trip assignment of a 15-minute alternative path diverted from a 10-minute shortest path is identical to that of a 105-minute alternative path diverted from a 100-minute shortest path if there is only one alternative for both path sets. In reality, it is unlikely that a significant proportion of trips would use the alternative (15-minute) path rather than the shortest (10-

minute) path; although it is likely that a significant portion of drivers would take the alternative (105-minute) path rather than the shortest (100-minute) path. The model, however, cannot discern these two different scenarios.

A more essential problem is that the model inherently does not sustain routes that overlap, a real-world situation. As is well known, the logit-based discrete choice model implies a choice mechanism known as the independence-of-irrelevant alternatives (IIA) axiom. Briefly, the axiom states that the ratio of the probabilities of choosing one alternative to that of choosing another is independent of the probability of choosing other available alternatives. This implies that the application of choice models with this axiom is most successful when the alternatives are perceived as being independent by the choice maker; otherwise, the independence axiom may be too strong an assumption to make.

Dial's algorithm inherently maintains the IIA axiom since it employs a logit-type formula to estimate the path utilization probability. At the same time, the algorithm generates "reasonable" paths that cannot avoid overlapping with each other. Restrained by this contradiction, the model cannot properly reflect the topology of a network.

The problem was first demonstrated by Schneider (25) and then by Burrell (23) by using a simplified three-route example. The example includes different network configurations in which Dial's method generates the same probability for each of the three routes in the network. As shown in Figure 12a, it is supposed that each of the three routes from A to B is of equal length, and each of the two routes between A and C is of negligible length compared with CB. Then it must be expected that, in reality, about half the traffic between A and B would use the route ADB and that each of the two routes going through C would carry about a quarter of the total. However, in Dial's method, if both paths through C are classed as reasonable, each of the three routes between A and B will carry one-third of the total traffic because of the rule that routes of equal length carry equal flows.

In the second network configuration, it is supposed that C is located halfway between A and B (Figure 12b). Each of the three routes from A to B are again equal, but each of the two AC routes are now equal to CB. It must be expected that, in reality, the proportion of traffic between A and B that uses route ADB will be about midway between one-third

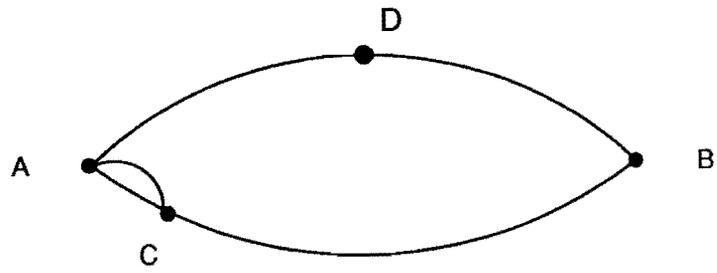


Figure 12a.

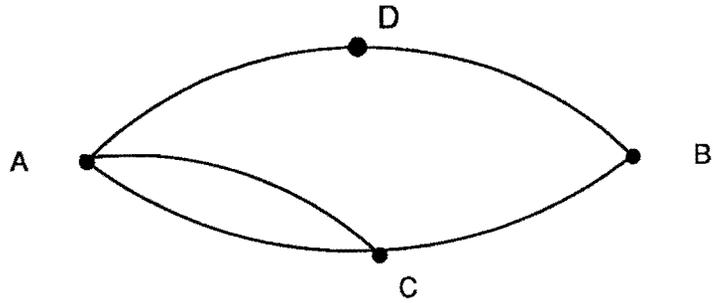


Figure 12b.

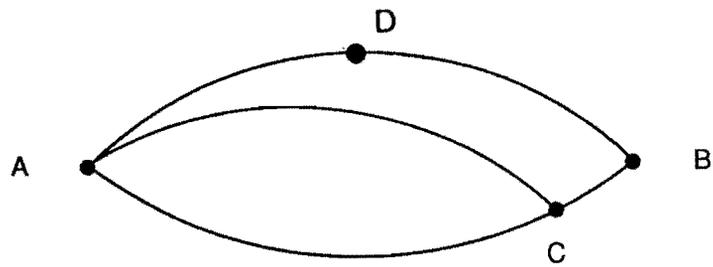


Figure 12c.

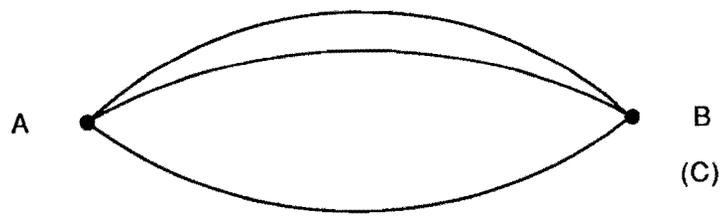


Figure 12d.

Source: Adapted from Burrell (23) and Schneider (25).

FIGURE 12. An Example of a Three-Route Network.

and one-half. Again, Dial's method loads one-third of the traffic on route ADB because of the rule that traffic divides equally among routes of equal length.

As shown in Figure 12c, C moves toward B. Each of the three routes from A to B is of equal length with CB being of negligible length. Then, it must be expected that, in reality, each of the three routes will carry approximately one-third of the traffic. Meanwhile, Dial's method will also load one-third of the traffic on each of these routes because their lengths are equal. It is evident that as C moves closer to B, Dial's method, consequently, produces more reasonable results to reflect this reality. As shown in Figure 12d, the configuration of three independent route networks is the most suitable for the method.

It is well recognized that in reality network links are usually correlated and routes unavoidably overlap each other. As in most assignment methods, Dial's algorithm defines distinct routes having some common links, and therefore, such routes (alternatives for trip maker) are not independent. In order to apply the discrete choice model to an assignment network, as proposed by Florian and Fox (24), a more stringent definition of independence would prohibit even common nodes except for origin and destination.

Although the algorithm that Dial presented is a single-pass assignment procedure, in theory it can also be reiterated and combined with the capacity-restraint procedure. Dial states:

The probabilistic assignment model has utility in coping with the capacity restraint problem. Because the model performs well 'on the margin', it is excellently suited for a time-stratified, 'incremental loading' technique. Such a technique would possess most of the good properties of a stochastic, time-dependent simulation model . . . (3)

Robillard developed a dynamic multipath traffic assignment model (26) based on Dial's algorithm. However, the capacity-restraint (impedance adjustment) procedure has not been incorporated in his research. Among others, Burrell (23) speculated that there may be a lack of robustness in Dial's method which arises from its definition of "reasonableness." This implies that instabilities may occur between iterations and that a small change of link cost may make a reasonable route become unreasonable and, thus, receive zero volume.

In summary, Dial's route-choice probability is derived from the logit-based discrete choice model. Thus, two drawbacks stemming primarily from the assumptions of the logit

model comply with Dial's algorithm. With the assumption of identical random error terms' variance among alternative routes, the model cannot accommodate the characteristic that route travel times are related to the respective lengths of alternative routes. With the assumption of the independence of irrelevant alternatives, the model cannot properly reflect the topology of the network. However, it is known that Dial's single-pass algorithm is very efficient in that it requires only a modestly higher amount of computational effort than the minimum path algorithm.

GUNNARSSON'S ALGORITHM

This method assumes that a driver traversing a network from an origin to a destination makes a choice of route at each of the nodes on the basis of the information available at successive nodes independently of any previous decisions. The driver's intention is to reach the destination by the route that seems to be the most suitable one at the point at which a selection is made (8). In other words, the route that a driver traverses from an origin to a destination is assumed to be a composite of a series of individual choices made on network nodes. The composition of these choices is based on the theory of the Markov chain which implies a stochastic process in which the future is determined from the present state(s) and is independent of the past.

A Markov chain may be regarded as a sequence of states through which a system passes at successive points in time. The probability of state x_j at time t (P_j^t) is equal to the sum of the product of the probability of the state x_i at time $t-1$ (P_i^{t-1}) and the transition probability (P_{ij}) over all possible states of x_i . It can be shown by an equation:

$$P_j^t = \sum_{i=1}^N P_i^{t-1} * P_{ij} \quad (3.4)$$

where: P_j^t = probability of being in the state x_j at time t
 P_i^{t-1} = probability of being in the state x_i at time $t-1$
 P_{ij} = probability of the transition from state x_i to x_j
 N = number of possible states of x_i

It is assumed that the probability of making a transition from one state to another depends

only on the present state occupied, not on any earlier history. Also, the transition probabilities P_{ij} are stationary and do not change over time.

To adopt the theory, several assumptions were made in Gunnarsson's algorithm. First, the driver has information at each node regarding the traffic conditions on the rest of the network and makes a choice of route at each node independently of any previous decisions. Second, the driver will accept a route via an adjacent node only if 1) this node is closer to the final destination than the relevant node at which a choice is made (has lower "potential"), and if 2) the prolongation compared with the minimum path is within a specific limit measured by distance differential or ratio. Third, the transition probability can be expressed in terms of the attractivity model as:

$$P_{ij} = \frac{A_{ij}}{\sum_{k=1}^N A_{ik}} \quad (3.5)$$

where: P_{ij} = the transition probability that a driver at node x_i will select a path via the adjacent node x_j with a pre-determined destination x_m
 A_{ij} = the attractivity for path ij (path from x_i to x_m through x_j)
= a probabilistic weight function of the resistance for link ij
 N = number of acceptable links departing from x_i

Conceptually, attractivity is a probability weight function that approximates the potential of attracting traffic onto a link. The measure is basically the same as the link weight in Dial's algorithm. For a comparison with the previous section, Equation 3.5 can be rewritten based on the notations of link weight:

$$P_{ij} = \frac{W_{ij}}{\sum_{k=1}^N W_{ik}} \quad (3.6)$$

where: P_{ij} = the transition probability that a driver at node x_i will select a path via the adjacent node x_j with a pre-determined destination x_m
 W_{ij} = the weight (potential attractivity) for path ij (path from x_i to x_m through x_j)
 N = number of acceptable paths from x_i

It should be noted that the denominator of the right-hand side in Equation 3.6 is the sum of all acceptable links that depart from the initial node, whereas in Dial's algorithm the denominator is the sum of all acceptable links that enter the end node.

Gunnarsson described his multipath algorithm by a series of matrix operations which are efficient in computation. A network consisting of N nodes x_1, x_2, \dots, x_n , each pair of which is connected by a link, is assumed. The distance from node x_i to x_j is d_{ij} . The distance relations for each node in the network are expressed by a matrix \mathbf{D} of elements D_{ij} . The matrix \mathbf{D} is an $N \times N$ square matrix in which the subscripts i, j have the values 1, 2, ..., N . One execution of the algorithm assigns a set of trips from a certain number of origin nodes to a destination node x_m . The algorithm consists of the following operations:

1. The minimum paths from all the nodes to the destination node x_m are examined. The shortest distances are expressed by a vector \mathbf{S} of element S_i , where i represents the origin nodes. The values S_i are then ranked in descending order to give a new vector \mathbf{S} and a corresponding vector of nodes, x_1, x_2, \dots, x_n .
2. A resistance matrix \mathbf{R} of elements r_{ij} representing the remaining shortest distance from x_i to x_m through x_j is defined. The matrix \mathbf{R} is an $N \times N$ square matrix, in which the subscripts i, j takes the values 1, 2, ..., N . Two steps are needed to construct the matrix \mathbf{R} :

- (i) the shortest distance is first placed in the matrix,

$$R_{ij} = S_i,$$

if x_j is on the shortest path from x_i to x_m , and

- (ii) if x_j is not on the shortest path, $S_j < S_i$ and $R_{ij} \leq w \cdot S_i$, where w is the acceptable prolongation factor for a route compared with the minimum path, then

$$R_{ij} = D_{ij} + S_i$$

3. The values of R_{ij} are then used to calculate the attractivity A_{ij} (potential of attracting traffic onto route ij):

$$A_{ij} = R_{ij}^{-\Delta},$$

where Δ is the calibrated parameter for different networks. After the values of A_{ij} are obtained, a matrix \mathbf{P} of elements P_{ij} is defined representing the transition probabilities according to Equation 3.5.

4. A matrix \mathbf{Q} of elements Q_{ij} representing the assigned link volumes is defined. Trips originating at the node x_i with destination x_m are first inserted in the diagonal elements (Q_{ii}) as positive values. For a row in the matrix, the elements Q_{ij} are calculated according to:

$$Q_{ii} = \sum_{k=1}^n Q_{ik}, \quad \text{if } Q_{ik} > 0, \text{ and}$$

$$Q_{ij} = -Q_{ii} * P_{ij}$$

After the treatment of a row, the elements are transposed so that the entering trips are equal to departing traffic; that is

$$Q_{ji} = -Q_{ij}$$

where the positive value represents entering traffic to a node x_i .

5. After the completion of the matrix, its balance can be checked by two means:
- (i) the entering and departing traffic at any node should be equal, i.e.,

$$\sum_{k=1}^n Q_{ik} = 0, \text{ and}$$

- (ii) the sum of the generated traffic should be zero, i.e.,

$$\sum_{k=1}^n Q_{ik} = 0$$

The whole procedure is repeated for the next column in the trip table until all the columns are exhausted.

Discussion

For the purpose of analogical analysis, these matrix operations were translated and reorganized into a flow chart similar to that of Dial's algorithm (Figure 11). As shown in Figure 13, one execution of the algorithm assigns a set of trips from a number of origin nodes to a destination node, i.e., a column of the O-D table. After a column of the O-D table is specified, the algorithm then builds an efficient-path bush rooted at the destination node. As mentioned in the previous section, two criteria can be used to search the efficient paths: 1) paths continually move further away from the origin node, and 2) paths continually move closer to the destination node. Dial's second algorithm uses only the first criterion to build the efficient-path bush. In contrast, Gunnarsson's algorithm uses the second criterion since it searches the efficient paths regressing from the destination to all other origins.

The algorithm consists of two major loops: path-searching and trip-loading. In the path-searching loop, the formula used to calculate path attractivity is different from that in Dial's algorithm. It is expressed in terms of attractivity (link resistance), whereas Dial employed an exponential-type (logit) formula to estimate the route choice probability. The mechanism of the trip-loading loop is basically the same with Dial's backward pass except that it proceeds in a forward manner.

Both Gunnarsson's (8,27) and Dial's (3) multiple path algorithms were developed about the same time and are similar in some features. Dial's algorithm searches multiple paths in ascending (forward) sequence and loads trips in descending (backward) sequence. Gunnarsson's algorithm, on the contrary, searches paths in descending sequence and loads trips in ascending sequence. Moreover, Dial's algorithm (second) builds an efficient-path bush rooted at the origin node and executes path-searching and trip-loading once for each row of the O-D table. Gunnarsson's multipath procedure builds an efficient-path bush rooted at the destination node, and executes the whole procedure once for each column of the O-D table.

Since Gunnarsson's algorithm is so similar to Dial's algorithm, an interesting question is: Will the algorithm have the same undesirable features as that of Dial's algorithm? Although in Gunnarsson's algorithm the logit-type formula is not used to estimate the route

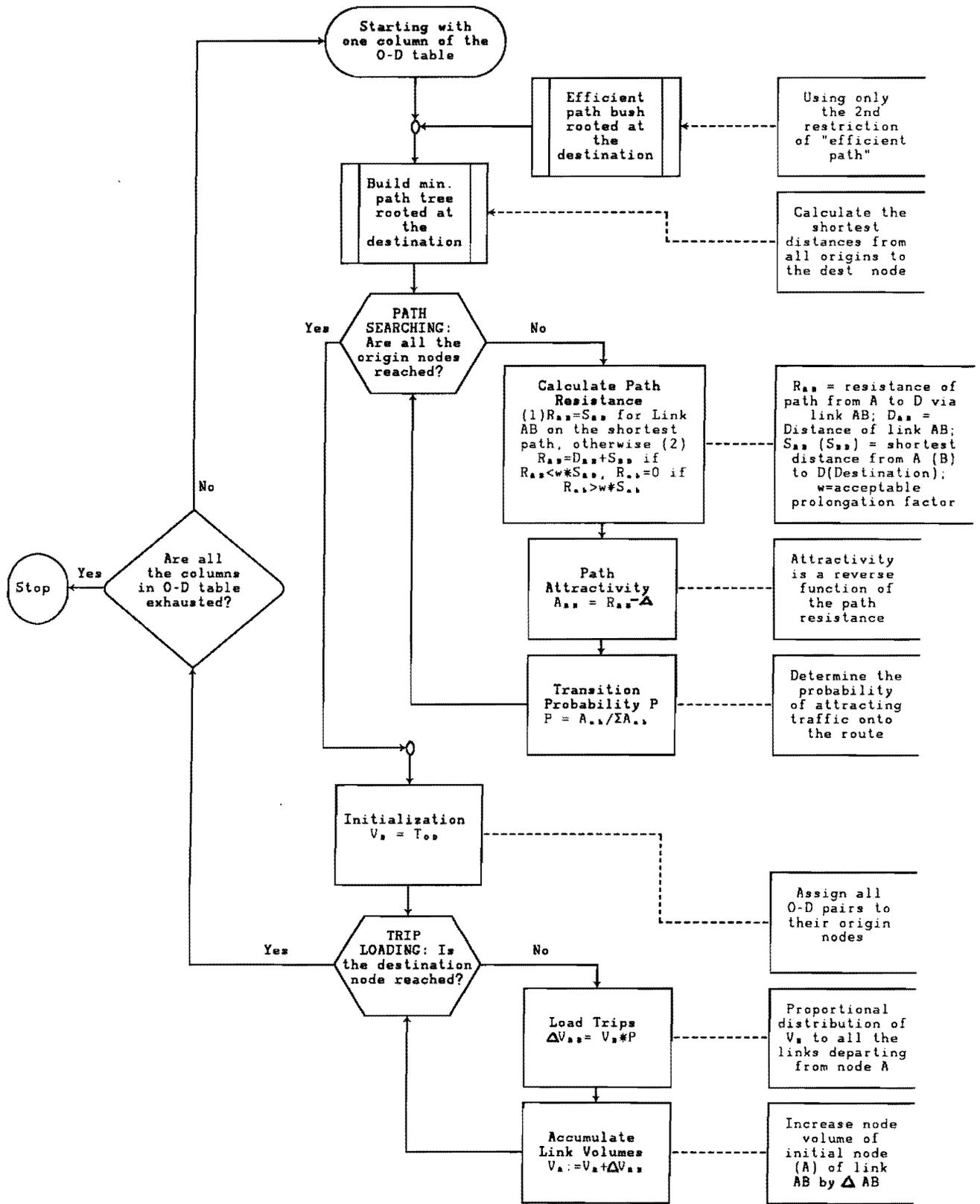


FIGURE 13. Flow Chart of Gunnarsson's Algorithm.

choice probability, the alternative routes' random errors have not been taken into account, either. However, the formula Gunnarsson proposed for estimating route choice probability seems more responsive to the length of a link. In an example in his paper (8), the transition probability is given as:

$$P_{ij} = \frac{R_{ij}^{-8}}{\sum_{k=1}^N R_{ik}^{-8}} \quad (3.7)$$

where: P_{ij} = the transition probability that a driver at node x_i will select a path via the adjacent node x_j with a predetermined destination x_m
 R_{ij} = the resistance for path ij
= the distance of path ij

Using the same example (comparison between a 10-minute and a 100-minute shortest path choice set) in the previous section, the probability of trips assigned to a 15-minute alternative path diverted from a 10-minute shortest path is no longer the same as that of a 105-minute alternative path diverted from a 100-minute shortest path if there is only one alternative for both path sets. The probability of using an alternative path in the first (10-minute shortest path) choice set ($15^{-8}/(10^{-8}+15^{-8})=0.04$) is lower than that in the second (100-minute shortest path) choice set ($105^{-8}/(100^{-8}+105^{-8})=0.40$). This, of course, is more responsive to what usually occurs.

The composition of route choices in Gunnarsson's algorithm is based on the theory of the Markov chain which implies that the future is determined from the present state(s) and is independent of the past. Moreover, it is obvious that all the possible present states are independent; otherwise, their probabilities could not be summed to represent the probability of the future state (see Equation 3.4). This implication is basically the same as the axiom of independence of irrelevant alternatives in logit model. By the same token, the model cannot properly reflect the topology of the network.

It can be concluded that the mechanism of the Gunnarsson algorithm is similar to

that of Dial's algorithm. The major difference between the two methods is the formula to estimate route choice probability. Gunnarsson's is expressed in terms of link resistance, whereas Dial's algorithm is expressed in a logit-type formula. Gunnarsson's algorithm inevitably has some of Dial's undesirable features. However, the matrix operations Gunnarsson uses to commence his algorithm is very efficient for computer application.

COMPARISONS OF THESE METHODS

Two types of multipath assignment techniques were discussed: path enumeration and path diversion methods. The most renowned path enumeration model is Burrell's algorithm, and Dial's and Gunnarsson's algorithms are exemplary path diversion models. Since the Gunnarsson algorithm is basically similar to Dial's, comparisons are primarily made between Burrell's and Dial's algorithm. Both Burrell's and Dial's models aim to appropriate the stochastic characteristics of drivers' behavior, but their approaches are quite different. In Burrell's procedure, multiple paths are obtained from different networks built from successive assignment processes by simulating link impedances iteratively. In Dial's procedure, trips from an origin to a destination are proportionally distributed among reasonable paths. The major differences between these two techniques are summarized in Table 1.

Unlike traditional assignment models, link impedances (travel times) are considered as a random variable in Burrell's model. Burrell's basic assumption to sustain travel time as a random variable is that for any specific link (route), every driver has his or her estimation of travel time. The random effect of the model is created by repeating stochastic (simulation) processes. On the other hand, Dial's stochastic assignment model is possibly derived from the logit-based discrete choice model. In logit models, the error terms in the utility function for different alternatives are assumed to be identically distributed with the same fixed variances. Hence, link impedances used in Dial's model to determine the share of trip assignment is, basically, a fixed variable.

Both methods have some deficiencies derived from their basic assumptions about the route travel time and route choice behavior. In Burrell's algorithm, it is assumed that link travel costs along a route are independent. The assumption conflicts with the reality that

Table 1
Summary of Comparisons between Dial's and Burrell's Algorithms

	Burrell's Algorithm	Dial's Algorithm
1. Possible Origin	PERT (Program Evaluation and Review Techniques)	Discrete Choice Logit Model
2. Basic Assumption	Link travel cost is a random variable since every driver (or user) has different perception of link travel cost	The diversion among reasonable routes can be represented by route-choice probabilities
3. Model attributes	Stochastic -- Simulation of link costs (time, distance, or/and cost)	Stochastic? Deterministic? -- Proportional distributions among reasonable routes
4. Route Searching Method	<ol style="list-style-type: none"> 1) Use all-or-nothing (A-O-N) assignment to generate minimum path trees based on randomized link cost 2) Multiple routes are generated based on the principle of link perturbation 3) Iterative Process is needed 	<ol style="list-style-type: none"> 1) Generate minimum path trees based on the all-or-nothing (A-O-N) algorithm 2) Estimate diversion probability based on length difference compared with the minimum path 3) Build reasonable path bush for each origin zone in turn 4) No routes are explicitly enumerated
5. Advantages	<ol style="list-style-type: none"> 1) Behavioral approach based on simulation 2) Iterative process suitable for congested network 3) Applications show better results than A-O-N 	<ol style="list-style-type: none"> 1) Behavioral approach based on random utility theory 2) One-pass process suitable for uncongested network 3) Applications show better results than A-O-N
6. Disadvantages	<ol style="list-style-type: none"> 1) Link travel costs along a selected route are assumed to be independent from one another 	<ol style="list-style-type: none"> 1) Random errors are assumed identical and fixed 2) Insensitive to network topology 3) Intensive calibration process

link travel costs in a selected route are usually correlated with one another. The major criticism of Dial's algorithm is that, with the assumption of the independence of irrelevant alternatives, the model cannot properly reflect the topology of the network. In other words, the model cannot discern overlapping routes and may over-assign trips to these routes.

Burrell's algorithm can be implemented both in a single-pass procedure and with the capacity-restraint procedure. Paths are enumerated by repeating simulations of link impedances for each origin zone (or a number of origin zones) in a single-pass procedure; paths are enumerated by repeating simulations of link impedances for each assignment stage when combined with the capacity-restraint procedure. In theory, Dial's algorithm can be implemented with the capacity-restraint procedure although the algorithm he proposed (3) is a single-pass procedure. In the procedure, trips are divided between all reasonable paths. The reasonable path is defined as one that does not trace back, and the share of trips assigned to each reasonable path is determined on the basis of total path cost difference between the reasonable path and the minimum path.

Strictly speaking, no real "simultaneous-multipath" routing techniques were built in either Burrell's or Dial's algorithm. It is obvious that Burrell's path-searching mechanism utilizes the minimum path assignment algorithm. In Burrell's procedure, the minimum path algorithm is used repeatedly to build minimum path trees after all the link costs are estimated. Dial's algorithm divides trips among all reasonable routes based on their total route cost when compared to the minimum path. Hence, the minimum path algorithm is also needed in Dial's algorithm to provide a basis for these comparisons. However, it is evident that the mechanisms of the two methods are more sophisticated than the minimum path assignment. Researchers claim that the methods produce better assignment results than the minimum path assignment (2,3,8,14,15,16,18,19,28).

IV. APPLICATIONS OF MULTIPLE PATH PROCEDURES

Among the various traffic assignment techniques (see Appendix A), applications of multiple path methods are the major concern in this chapter. Since there are several articles involved, this chapter is organized as a listing of annotated articles. Each item consists of a summary of the content and discussions of the merits of the article based on the authors' interpretation.

APPLICATIONS OF BURRELL'S ALGORITHM

Before Burrell's algorithm was developed, the concept of variations in link travel times had been considered by Von Falkenhausen (29) who suggested an assignment based on log-normal distribution of link travel times. Burrell used an assignment based on discrete-uniform distribution and extended the concept so that paths are enumerated by having a different set of link travel times for each origin zone. The method is particularly welcome for its simple implementation. It can be implemented in a single-pass procedure or with the capacity-restraint procedure. Applications of the method have been carried out mostly in England and Canada (14,15,16,17); a similar effort also has been tried in the United States (18,19).

1. **Burrell, J. E., "Multiple Route Assignment and Its Application to Capacity Restraint," Proceedings, 4th International Symposium on the Theory of Traffic Flow, Karlsruhe, June 1968.**

The paper discusses a multiple route model in which it is assumed that the person making a trip in a network does not know the actual journey time on each link of the network, but with each link a "supposed" time is associated which is drawn at random from a distribution of times having as its mean the actual (observed) link time. For reasons of computation, Burrell used eight possible values to represent the link time distribution. He also mentioned that, in principle, one might wish to use a normal distribution of link speeds; but with only eight values it is not possible to represent the tails of a normal distribution. It was decided in practice to use a simple rectangular distribution with each of the eight possible values

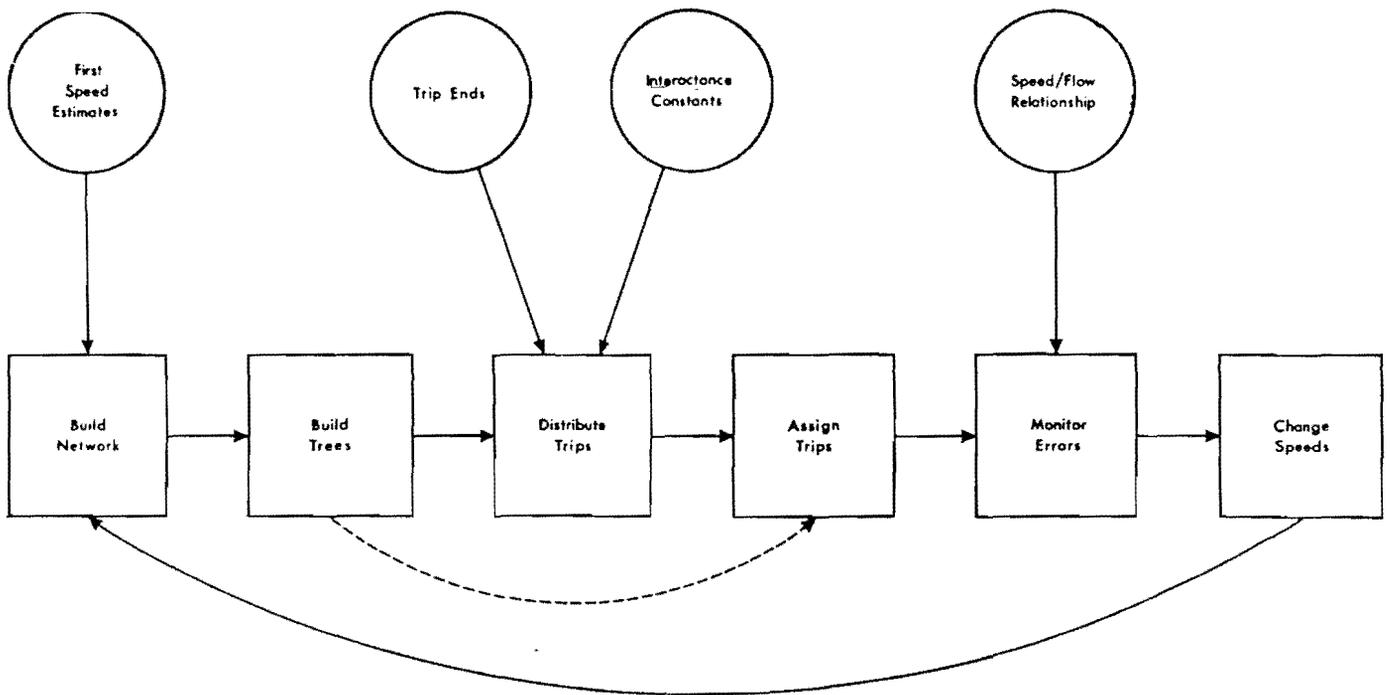
having an equal chance of being chosen.

The spread of link times at about the average was measured by the mean absolute deviation, that is, simply the mean absolute deviation from the average, ignoring the sign. The spread of times for each link was chosen so that the ratio of mean absolute deviation to actual link time was the same for all links in any particular assignment. The ratio is referred to as the diversion factor. This seems to indicate that Burrell used only one diversion factor for all the links in a network. Later in Mason's (16) and Dalton and Harmelink's (14) application of the method, different mean unit cost deviations were permitted in the program for various link classes. Their application is more realistic in that urban arterials and access-control freeways are supposed to have different mean deviations of link travel times.

The route searching mechanism is based on the conventional minimum path algorithm. Multiple routes are presumed to be "perturbed" (see Chapter 3, Burrell's algorithm) since the link times are random and not fixed. Only one tree was built for each origin zone since tree-building requires more computer time than other mechanisms. However, Burrell recognized that it is possible to use the multiple route procedure in a more advanced form so that the trips from each origin zone can be divided into several equal parts, each of which has its own tree built from a different set of supposed link times. Burrell claimed that the computer time required is very little different from that required for conventional minimum path method. The only extra time that is required for calculating the supposed link times from actual link times is quite small.

The procedure can be used for making assignment in unrestrained situations or where capacity restraint is required. According to Burrell, one of the most valuable applications of the multiple route method is found to be in capacity-restraint procedures. However, he does not present any further evidence to support this statement. A flow diagram of the sequence of operations is shown in Figure 14. In the diagram, circles are used to represent data and squares for computer process. In addition to the standard processes of Build Network, Build Trees, Distribute Trips, and Assign Trips, there are two further processes: the Change Speeds and Monitor Errors programs.

The Change Speeds program produces a new speed from each assignment for each



Source: Burrell (2).

FIGURE 14. Burrell's Capacity-Restraint Procedure Flow Chart.

link which is used for the next iteration. This is done by connecting the speed-flow point R from the previous assigned flow to the origin and finding the point Q where this crosses the speed-flow line (see Figure 15). The new link speed corresponds to a point S (called the intercept) such that $RS/RQ = p$, where p is constant, the same for all links. It can be chosen to take any value between 0 and 1.

The purpose of the Monitor Errors program is to monitor the convergence process. The program was designed to produce the following information:

1. A histogram of number of links by flow/capacity ratio.
2. A listing of all links for which the flow/capacity ratio lies outside preset limits.
3. An analysis of assignment error which reflects the difference for each link between the assignment produced and the capacity of the link at the same speed. The error for the whole system is defined as:

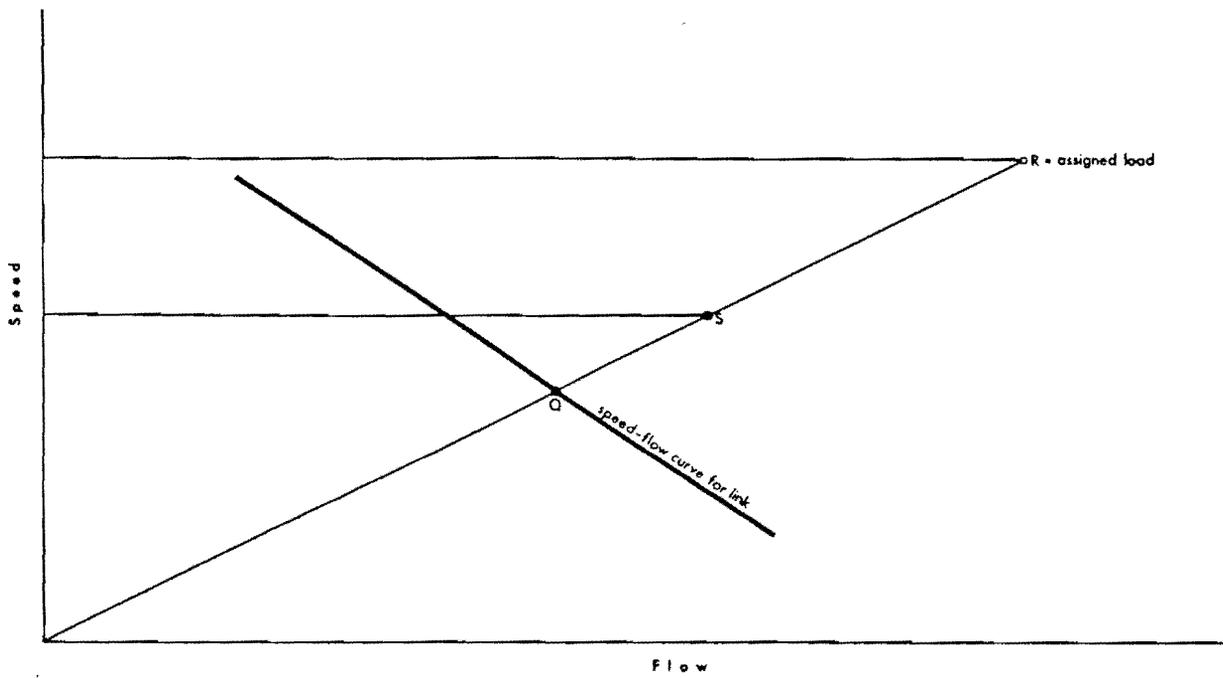
$$\% e = \frac{\sum \left| \frac{\text{Flow-Capacity}}{\text{Capacity}} \right|}{\text{Number of Links}} \quad (4.1)$$

4. The number of links for which the speed-flow point lies on the opposite side of the speed-flow line than it did in the previous assignment.

The last information is found to be an especially valuable indicator of the working of the convergence process.

The multiple route method was combined with the capacity-restraint procedure and then was applied to a possible future London network. In the application, two main parameters, the diversion factor and the intercept factor, could be chosen. They were presumed to have a large effect on the rate of convergence. Several combinations of the two parameters were tested. The overall best value of error reduction acquired was the combination of diversion factor = 0.4 and intercept factor = 60%. Nevertheless, it is unknown whether, and to what extent, the optimum parameter values varied between different network.

The two parameters were used throughout the assignment procedure. After the fifth



Source: Burrell (2).

FIGURE 15. Speed Adjustment Method in Burrell's Procedure.

iteration, the remaining error on the most important roads was only some 10 to 15 percent. The author thought it not worthwhile to take the iteration further as errors from other sources (e.g., land uses and car ownership predictions, distribution functions, etc.) were probably of this order of size; and, thus, increasing the accuracy of the assignment process itself would not substantially improve the overall accuracy of the final result.

The author concluded that perhaps the most important feature of the procedure was its simplicity. He states:

The elementary device of having for each zone of trip generation, a set of link times which differs slightly and in a random way from the set of link times for each other zone, proved to be a powerful method of obtaining multiple routes between pairs of nodes. This in turn enabled rapid convergence to be achieved in situations where capacity restraint is necessary.

There seems to be a missing linkage between these two sentences of conclusion. The elementary device here apparently refers to Burrell's single-pass procedure; and no sufficient evidence shows that rapid convergence was achieved when the method was combined with the capacity-restraint procedure.

2. Mason, P. R., "Multiflow Multiple Routing Procedure," Traffic Engineering and Control, Vol. 14, 1972, pp. 280-282.

The concept of "multiflow" is similar in principle to the multiple route procedure developed by Burrell with minor variations in detail. The multiflow multiple routing procedure simulates the random variation in drivers' estimates of cost by repeated randomization of link lengths, each randomization being followed by the generation of a set of minimum routes. The link lengths are randomized by selecting for each link a random point from a discrete link length distribution whose mean is the coded link length.

Compared with Burrell's procedure, a major advancement of the multiflow multiple routing procedure is that the variances of the link length distributions are proportional to the coded length (see equation 3.2 in Chapter 3). In this way, the degree of deviation from the coded length which is applied to a section of road is independent of the number of links into which it is divided. It was also suggested that other cost parameters, such as time

and/or monetary cost, could also be randomized in a similar way in order to simulate drivers' differing values of time and cost.

According to the author's observation, the multiflow multiple route assignment program has been used successfully on more than 25 transportation studies over a period of four years (the paper was published in October 1972). In addition, experience has shown that a multiple route using only one tree or vine per origin zone, with about the same computer cost as an all-or-nothing assignment, will produce far better results. Greater accuracy may be expected by increasing the number of trees or vines per origin zone. The method recommended that one or more trees or vines be built per origin zone, each tree or vine being based upon a single cost formulation and a single randomization of all lengths in the network.

The author concluded that the method is of particular value in connection with the capacity-restraint procedure, since speed/flow convergence is more rapid when trips are divided adequately between competing routes. He states:

Multiflow multiple routing may be used in an incremental loading capacity restraint, but it has the important advantage that it is possible to make a non-incremental capacity restraint assignment to converge, and to produce an assignment in which speeds and flows are compatible. Since multiflow multiple route assignments offer considerable advantages over all-or-nothing assignments, there would appear to be few occasions where the use of all-or-nothing is justified.

In theory, the method can be used with not only the incremental but also the iterative capacity-restraint procedure. Moreover, sufficient evidence to support this conclusion and on what basis the speed/flow convergence is evaluated have not been shown in the report.

3. Dalton, P. M. and M.D. Harmelink, "Multipath Traffic Assignment: Development and Tests," Report No. RR189, Research and Development Division, Ontario Ministry of Transportation and Communications, Downsview, Ontario, 1974.

Based on the concept of Burrell's algorithm, a multipath assignment technique was

incorporated into the capacity-restraint procedure. Since 1968, the authors have continued to develop and test the method in Ontario. The initial emphasis was on producing realistic assignments from survey data and a new program using an IBM 360/65 computer. The final stage of the development was to consolidate capacity-restraint procedures and to conduct an extensive series of tests on the Toronto network in order to evaluate the different techniques. The tests were conducted with and without the multipath technique. The program may be broken down into two sections which, according to the authors, are quite independent of each other and may be used as such or together. They are the multiple route assignment and the capacity-restraint procedure.

Multiple Route Method

The multiple route method is based primarily on the concept of Burrell's algorithm. Several points extended from Burrell's origin procedure are outlined below:

Number of Trees

To obtain the exact split of all possible paths between two nodes, it would be necessary to build every possible combination of link costs. In a network with "n" links, this would mean building 8^n trees for each origin zone (assuming each link has eight variations). This is obviously unfeasible in practice. In Burrell's algorithm, a sample of all possible trees is taken by using random numbers to choose the link cost, and only one tree is built for each origin zone. As such, the generation of multiple routes relies on the principle of the "link perturbation" method. In this program, more than one tree per origin zone was recommended. However, no distinct number of trees per zone was specified. Only a principle of delineation was outlined (page 6 of the report):

The program works on the principle that any set of trips assigned at one time should be an insignificant part of the total volume on any link used. It follows that it is desirable to split the volumes up in such a way as to achieve the greatest possible efficiency of computation time.

The number of trees is related to the position of an imaginary cordon.

Imaginary Cordon

For the random selection method to be valid, the volumes assigned to any one tree must be small enough not to have a significant effect on the total assignment on any link. This was found to be true for most links in the network when building one tree per zone except on the links immediately surrounding the origin zone. This meant that, in order to avoid a large block assignment to a few links, it was necessary to build several complete trees over the whole network for each zone. Since the area affected around each zone was small, it was extremely wasteful of computer time to build complete trees each time. The authors then suggested the "imaginary cordon" method to improve the efficiency. An imaginary cordon was drawn around each origin zone at a specified travel time value and several trees were built inside the cordon for each complete tree. Since zones which have a large number of trips originating from them will have a greater effect on the assignment than other zones, the number of complete trees built for each zone may be varied according to the number of trips originating in the zone.

Link Costs and Link Cost Distributions

The link costs employed in the program may be time, distance, or any linear function of the two. Like the multifold multiple routing procedure (16), the values specified in the link cost distributions are proportionate deviations from the link values specified in the network. Also, rectangular distributions instead of normal distributions were employed for computational efficiency. Sixteen link classes permitted in the program were put into six or fewer groups, and separate values of the mean deviation were specified for each group. Normally the values are graduated through the different types of roads with urban collector streets having the highest and rural freeways the lowest. The deviations are also varied according to the length of the link. This may be done in either of two ways:

1. The deviations may be specified for each of any number of ranges of link costs with a maximum permissible mean deviation of 50 percent.
2. One cost may be specified at which any link with the value will have a mean deviation of 40 percent. All link costs below that value will then use the value of 40 percent while those above are assigned on the basis that the deviation

on any route should be independent of the cost. For example, a link at double the cost will have $40/\sqrt{2}$ percent mean deviation; and one at four times the cost will have 20 ($40/\sqrt{4}$) percent mean deviation.

The authors recommended that the second method is easier to use and involves specifying only one value for each group of links.

Options for Imposing Turn Penalties and Turn Prohibitors

The program provided options for imposing turn penalties or turn prohibitors to avoid trivial and unrealistic alternative routes. Care must be taken in placing turn penalties to insure that every zone can be reached.

Capacity-Restraint Procedure

The authors outlined three basic areas in which the program contains options in the methods used, and these may be combined to produce quite a wide range of techniques for the overall capacity-restraint procedure. They mentioned that some of these techniques are identical to methods which have been in use for some time, and there were variations of those methods available as well as new techniques. The three basic areas of choice are:

Methods of Assignment

Two choices of assignment algorithms are available in the program. They are multipath and minimum path (all-or-nothing) assignment algorithms. The capacity-restraint section of the program may also be used on its own to adjust the link speeds on a network previously assigned by any method provided that the assigned network tape is in the correct format.

Methods of Speed Adjustment

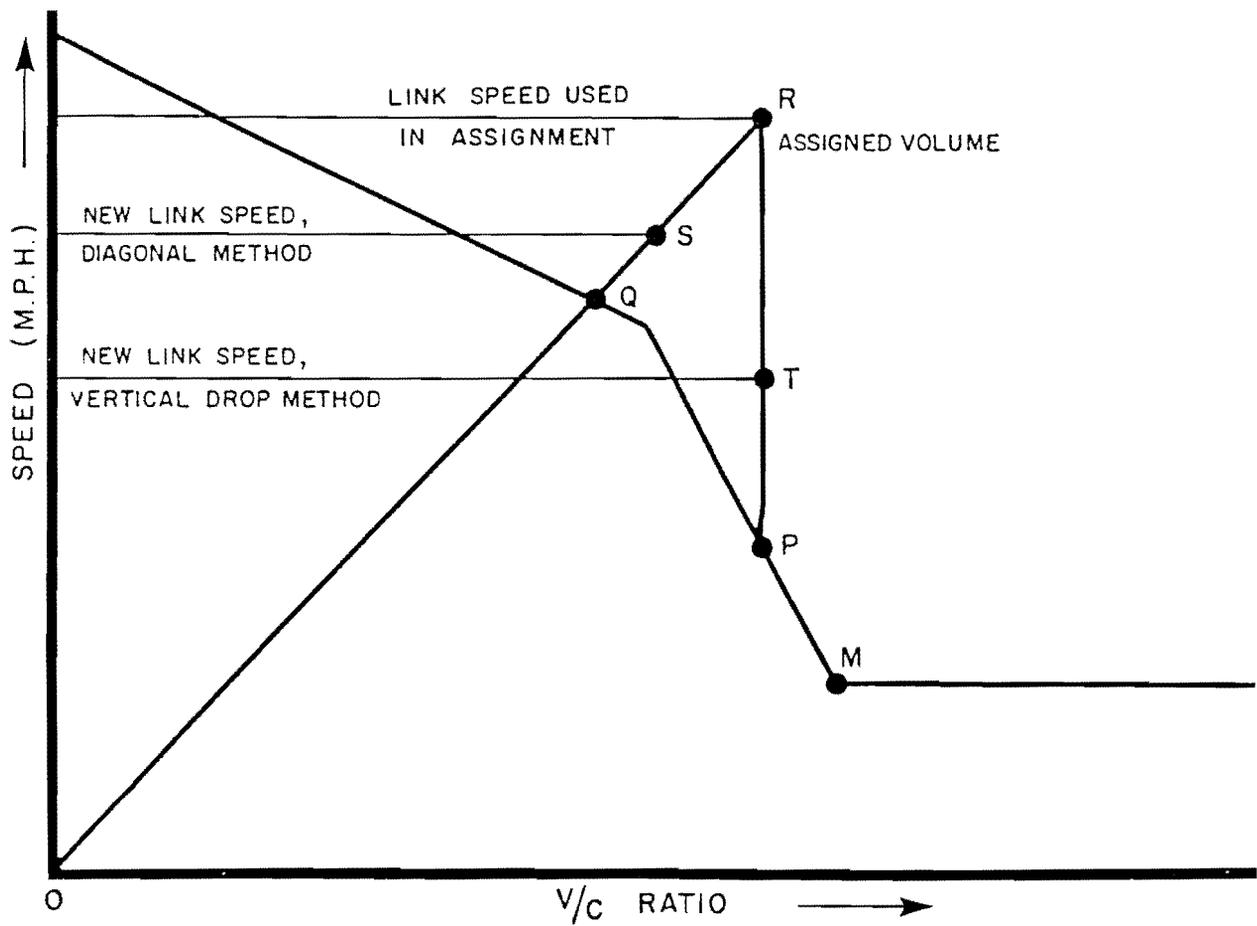
The program offers two methods of speed adjustment: diagonal adjustment and vertical drop. These two methods are similar in that they both consist of joining the assigned speed-flow point to a point on the capacity function curve and then arriving at a new speed by moving a fixed proportion of the distance along this line toward the capacity function. The proportion is called the damping factor (same as the "intercept" in Burrell's procedure) and

is an input parameter which is the same for all links. The difference in the two methods is in the choice of point on the capacity function. The diagonal adjustment method takes the point where the capacity function (speed/flow curve) intercepts a line drawn from the origin to the assigned speed-flow point. This is basically the same method used in Burrell's procedure. The vertical drop method takes the point on the capacity function which has the same V/C ratio as the assigned speed-flow point. Both methods are shown in Figure 16 for the same damping factor. The damping factor may take any value between zero and one. The additional network inputs required for both methods are the design capacities of each link (coded into the network when it is built), a capacity function for each link class, and a capacity index used in the network. The capacity functions consist of a series of speeds for specified V/C ratios, and the relationship is considered to be linear between these points.

Use of Adjusted Speed and New Assignments

The re-assignment can be used for either of two basic capacity-restraint methods: iterative and incremental assignment techniques. Basically, in the iterative process, the speeds are adjusted according to the specified parameters, the new link speeds are then used to produce a completely new assignment, and the previous link volumes are ignored. The process is repeated until an acceptable speed-flow relationship is achieved on all links. In the program, the incremental assignments are designed in two ways. The first is called "true" incremental assignment where the speed adjustments are made according to the volumes assigned so far; that is, the assigned volumes have not been expanded to 100 percent at each increment. The second incremental assignment is based on the method outlined by Smock (12) in which the network is kept fully loaded at each stage, and the incremental effect is achieved by assigning a proportion of the trips after each speed adjustment.

The authors then conducted several network testings which include unrestrained networks and capacity restrained networks. The purpose of the unrestrained network tests was to see whether the multipath method produced significantly better results than a minimum path assignment. Three series of capacity restrained network tests were carried out to evaluate the capacity-restraint procedure using the network for the 1969 metropolitan



Source: Reference Dalton and Harmelink (14).

FIGURE 16. Speed Adjustment Method in Dalton and Harmelink's Application.

Toronto evening peak hour. These tests were primary comparisons among minimum path iterative, multipath iterative, minimum path incremental, and multipath incremental methods. A combination of multipath iterative and multipath incremental was also tested.

Several conclusions drawn by the authors based on their network testings are worth noting. These include (page 23 of the report):

1. In all instances for single assignments, restrained or unrestrained, the multipath assignment technique produced better results than the minimum path according to three criteria: (a) closeness to observed volumes, (b) closeness to speed-flow relationships of the links, and (c) the portion of links with assigned volumes in extreme excess of capacity. The improvement was only slight where the network was calibrated for a minimum path assignment, but it was significant with an uncalibrated network or when calibration was carried out with the multipath assignment.
2. In order to obtain the best results possible with a multipath assignment it is necessary to pay particular attention to turn prohibitors and penalties in the network.
3. The extra cost required to run a multipath assignment instead of minimum path is not likely to be significant in any type of study. Most of the benefit of the multipath assignment occurs when building just one tree per zone at an increase in running time of about 10 percent over minimum path. The marginal benefit of building additional trees diminishes very rapidly and in none of the tests carried out did the running time of the multipath exceed that of the minimum path by more than 200 percent.
4. To obtain reasonable results by an iterative method of capacity restraint, it was found that using to use the multipath technique was essential.

5. The iterative method of capacity restraint with multipath assignment produced significantly more realistic link speeds than any of the incremental assignments. This result was further confirmed by comparisons made against journey time and delay studies.

It is well known that transportation planning is an art as well as a science. No single assignment technique is better than other techniques in all instances. It should be noted that these conclusions were drawn based on the network testings conducted by the authors. For example, the fourth conclusion may seem to be too imperious, and in some instances the incremental multipath assignment may produce assignment results as reliable as the iterative multipath assignment.

The overall conclusion made by the authors is that the multipath assignment technique can be used to advantage for most types of network with or without capacity-restraint assignment. He states: "In order to produce realistic link speeds by capacity restraint it is necessary to use the iterative method with multipath assignments, but otherwise the method of capacity restraint has little effect on the results." Again, this statement is presumptuous; other applications (2,16) show that the incremental multipath assignment produces reliable results. In general, the paper is one of the very few traffic assignment practices that is well documented.

4. Wildermuth, B. R., "The Use of a Multiple Routing Technique for One-Pass Capacity Restraint Assignments," Traffic Quarterly, April 1972.
5. Le Menestrel, H. and B. R. Wildermuth, "Toward an Efficient Highway Service Estimator for Congested Networks," Highway Research Record, No. 392, 1972.

These two papers are primarily applications of the same method. The method is a combination of the multiple routing algorithm with Schneider's capacity-restraint assignment process. Schneider's method, which was developed for the Chicago Area Transportation Study (July 1960), is a one-pass incremental process whereby the fraction of the total traffic demand loaded during each increment is the total demand from one origin centroid (see Chapter 2, Incremental Capacity-Restraint Procedure). Schneider's procedure is based on

the following assumptions: 1) each unit of traffic seeks to travel along the best available path, 2) the addition of each unit of flow would reduce the speed of travel on a link, and 3) a different best path situation exists for each new unit of traffic demand desiring to move through the network.

In theory, the one-pass incremental process updates link travel times for the whole network after trips from an origin zone to all other destination zones (the demands of one row of the trip table) are loaded. In actual application of the method, a number of origin centroids are selected at a time, and link travel times are updated after trips from these origin centroids are loaded. As previously mentioned in Chapter 2, this type of incremental assignment has a disadvantage in that the variability in results depends upon the order of centroid selection. This indicates that there may be a lack of robustness of the model.

Applications of the method to a 100-centroid network and a 165-centroid network were presented. The authors found that results obtained with the multiple routing one-pass incremental capacity-restraint procedure indicated a level of accuracy which is equivalent or superior to that obtained with other all-or-nothing incremental procedures. The authors concluded that the method provides a flexible and economical tool for the estimation of highway service measures on congested networks. However, it is speculated that more testings and analyses on larger networks should be carried out to support this conclusion.

APPLICATIONS OF DIAL'S ALGORITHM

Since its appearance in 1971, Dial's multiple path assignment model has drawn attention from researchers and practitioners of transportation planning. Extensions or modifications of the algorithm have been proposed: Robillard (26) generalized the method to consider dynamic input flows; Tobin (30) suggested including trip makers' perceptions in the model; Trahan (31) refined the computational steps of the algorithm; Florian (32) suggested an indirect way of introducing congestion effects; and Van Vliet (33) presented an attempt at finding equilibrium flows using a similar procedure. Calibration of the model has been considered by Robillard (34), Fisk (35), and Edwards and Robinson (28). Dial's assignment model also stimulated many discussions among researchers: Schneider (25) and then Burrell (23) discussed some undesired features of the model; Florian and Fox (24) examined its

probabilistic origin and the consequent problems. Finally, most of the past research examines the algorithm or modified algorithms by using hypothesized simple network examples; very few papers attempt to apply the method to actual networks or to evaluate the method in comparison with alternative approaches.

1. **Edwards, J. L., and F. O. Robinson, "Multipath Assignment Calibration for the Twin Cities," Transportation Engineering Journal of ASCE, Vol. 103, November 1977, pp. 791-806.**

The authors found that limited experience exists with the use of Dial's multipath assignment model in actual planning applications, and few guidelines can be found for the proper selection of model parameters (calibration) although the model has stimulated much interest since its appearance. This paper is primarily based on a research project undertaken at the University of Minnesota and jointly supported by the Twin Cities Metropolitan Council and the Minnesota Highway Department to investigate the use of the probabilistic multipath option in the UPTS (UMTA Transportation Planning System) program UROAD (36). According to the authors, the objectives of the research were threefold: 1) to calibrate the probabilistic multipath model using the 1970 highway network for the Twin Cities together with recorded ground count volumes from the same year, 2) to investigate the sensitivities of the model outputs to changes in input parameters (the diversion parameter θ and link impedances), and 3) to establish general guidelines for future use of the model in the Twin Cities metropolitan area. Several aggregate measures were selected to determine the goodness of fit to recorded ground count volumes including volume-to-ground ratios by route, facility type, screenlines and cutlines; and percent root mean square error and percent standard deviation by volume group and facility type.

For most logit-based models, the maximum likelihood principle could be used to estimate the parameters associated with them. In Dial's model, the parameter θ can be chosen by the transportation planner to calibrate the model with some degree of precision by using the maximum likelihood method (34). However, the value of the diversion parameter used in this research ($\theta = 0.002$) was adopted from the experience gained from a medium-sized city in Wisconsin which the authors assumed was similar to the Twin Cities

situation. This calibration approach is considered as an analogical method based on sensitivity analyses rather than the maximum likelihood method.

The research results indicated that the model produces an assignment which most closely approximates the recorded volumes when 1) link impedances are composed of a linear combination of time and distance (weighted more heavily in favor of distance), and 2) a value of 0.002 for the diversion parameter θ is used together with the model. The authors also found that it generally produced a better assignment than the all-or-nothing model. The model exhibited some sensitivity to changes in the diversion parameter θ , but model sensitivity to changes in the impedance function seemed much stronger.

2. Robillard, P., "Multipath Traffic Assignment with Dynamic Input Flows," Transportation Research, Vol. 8, 1974, pp. 567-573.

In this paper the author attempted to relax the assumption in most transportation planning models that the stationary traffic flow, where the input flows are assumed, is constant through time. Robillard noted that the dynamic aspect of the traffic flow is the key to a better understanding of the transportation phenomena.

The study presents an approach to the realization of this goal by adapting Dial's probabilistic multipath assignment model to the case of non-stationary input flows. Given non-stationary input flows, the assignment model was designed to evaluate the flow distribution through time for each point of the transportation network. According to the author, the model was expected to serve in studying the formation and the evolution of congestion and to give a more precise image of reality. However, no capacity restraints exist on the links in the model.

The paper is one of the pioneering research attempts to model the real-time traffic pattern. In general, it is primarily a theoretical development rather than an algorithmic model development. No testing of the model to a real-world network has been shown in the report.

3. Tobin, R. L., "An Extension of Dial's Algorithm Utilizing a Model of Tripmakers' Perceptions," Transportation Research, Vol. 11, 1977, pp. 337-342.

The purpose of this paper was to extend Dial's algorithm by utilizing a simple model of trip makers' perceptions and behavior. The author proposed three traffic assignment models which, according to the author, were computationally equivalent to Dial's model and could be implemented by making minor changes in his algorithm. Also, an important advantage these models have is that they can be calibrated through the study of trip makers' behavior on a limited set of trips rather than through the study of origin and destination patterns and link flows on a large network.

The first model is based on the likelihood that trip makers will choose a path; the second is based on the likelihood that trip makers will arrive at an intersection on a particular link; and the third is based on the likelihood that trip makers will turn onto a particular link at an intersection. The model, based on path likelihoods, contains Dial's model as a special case in its general form and attempts to eliminate some undesirable properties of Dial's model. However, like Dial's model, it generally allocates a disproportionate number of trips to portions of a road network in which there are a large number of choices of minor variations in route; that is, it does not reflect the topology of the network. The model based on arrival likelihoods and the model based on turn likelihoods both were developed to overcome this shortcoming. These two models differ in the order in which the nodes and links are processed and consequently have different interpretations in terms of trip maker behavior.

One primary assumption of these models is that the trip maker cannot always identify the minimal time path and may choose a path which is not of minimal time. This is interpreted by a trip maker perception model in which the parameter δ (coefficient of discrimination) such that δ multiplied by a function of the path gives the magnitude of the largest deviation from a minimum time path a trip maker would choose. However, details of calibrations of the parameter and the models have not been described. Also, no application of these models to an actual network is presented.

FINDINGS ON THESE APPLICATIONS

The main concern of this chapter is the application of two multiple path assignment models: Burrell's and Dial's algorithms. Burrell's algorithm is a path enumeration model which

reiterates the assignment procedure with variable link impedance. The method can be implemented in a single-pass procedure or with the capacity-restraint procedure. It is welcome for its simple implementation, particularly for networks that are already calibrated for the all-or-nothing or the capacity-restraint assignment since only minor modifications are needed. Several actual network applications of Burrell's algorithm have been carried out mostly in England and Canada (14,15,16,17); a similar effort also has been tried in the United States (18,19).

The merit of Burrell's algorithm is the concept of variations in link travel times. The concept has been considered by Von Falkenhausen (29). Burrell (2) extended the concept so that paths are enumerated by having a different set of link travel times for each origin zone (i.e., Burrell's single-pass procedure). Enhancements also have been done in the following applications of the model: 1) Mason (16) proposed that the variations of link travel time be proportional to the link length, and more than one tree can be built for each origin zone; and Dalton and Harmelink (14) suggested a method of delineating an imaginary cordon to improve the computational efficiency and options for imposing turn penalties and turn prohibitors to improve the assignment result.

Dial's multiple path assignment model has drawn attention from researchers and practitioners of transportation planning since its appearance in 1971. However, most of the past research examines the algorithm or modified algorithm by using hypothesized simple network examples; very few papers attempt to apply the method to actual networks or evaluate the method in comparison with alternative approaches. Edwards and Robinson's research (28) is based on an application of the probabilistic multipath option in the UTPS (UMTA Transportation Planning System) program UROAD. Robillard's (26) and Tobin's (30) research are extensions of Dial's algorithm without actual network applications in their papers.

A practical problem with Dial's algorithm is that the calibration of the diversion parameter θ could be labor intensive and frustrating. The calibration procedure not only needs to solve sophisticated non-linear maximum likelihood equations (35), but also requires substantial traffic data such as traffic volumes and travel times on the network links. Alternatively, applications of the model can be calibrated by an analogical method, by using

the already calibrated parameters from a network similar to the one in question.

Most of the papers discussed above indicate that the multiple path assignment techniques, both Burrell's and Dial's algorithms, are obviously advantageous compared to the minimum path (all-or-nothing) assignment. They draw this conclusion for the most part based on the comparison between the assigned volumes and the ground counts. Some papers even assert that the multiple path assignments can achieve faster convergence of the flow/speed relationship than the minimum path assignment. Burrell states " . . . it is found that the convergence of iterations is obtained much more quickly using multiple routes, than using single routes" (2). Tobin states: " In cases where congestion is important, Dial's model can be used in place of an all-or-nothing model in many of the iterative capacity restraint methods to give faster convergence to an assignment" (30). They believe that, in theory, the capacity-restraint procedure should decrease as the number of iterations increases. It is speculated, however, that more evidence is needed to support these statements.

V. SUMMARY OF FINDINGS AND RECOMMENDED FUTURE RESEARCH

Generally, the underlying assumptions of most multiple path assignments are obviously more realistic than that of the minimum path assignment. It is well recognized that travel time is not the only motivation for drivers' choice of route. The motivation is often a matter of safety, driver comfort, or a variety of other considerations and not just the minimum travel time. Moreover, the travel time itself is not a fixed element since temporal variations and measurement errors are unavoidable in actual networks.

In summary, the multipath (stochastic) assignment techniques were developed in recognition of one or more of the following:

1. The impedance (travel time) on a link, or series of links, is a random variable and not a fixed value.
2. For most zone pairs, there are a variety of paths for which the travel times are similar.
3. Drivers do not, and cannot, differentiate between paths for which the travel time (impedance) differs by a small amount.
4. Where drivers have a multiplicity of choices, the specific route taken will vary depending upon conditions encountered on a given trip.

However, no multiple path assignment model has proven capable of satisfying all these four requirements. Burrell's algorithm regards link travel time as a random variable, but it does not generate a number of similar cost paths between a pair of centroids and assign a portion of the trips to each of them. Dial's algorithm assigns trips proportionally to all "reasonable" paths, but the model cannot properly reflect the topology of the network.

Most multipath assignment techniques are based on one of two approaches: either the path enumeration or the path diversion. Path enumeration models reiterate the assignment procedure with variable link impedance inputs. Burrell's algorithm is a typical path enumeration model where the link impedances are assumed to be randomly distributed to account for errors in the driver's perception in link travel time. Path diversion models assign trips to alternate paths without repeating the assignment procedure. The most noted path diversion model is Dial's algorithm. Dial's technique originated from logit-discrete

choice theory in that each "reasonable" path between a particular O-D pair is assigned a portion of the trips according to route-choice probability.

The literature review indicates that these multiple path algorithms can be incorporated into the capacity-restraint procedure, either iterative or incremental. Burrell's algorithm can be implemented either in a single-pass procedure or with the capacity-restraint procedure. Paths are enumerated by repeating simulations of link impedances for each origin zone (or a number of origin zones) in a single-pass procedure; paths are enumerated by repeating simulations of link impedances for each assignment stage when combined with the capacity-restraint procedure. Burrell's algorithm is recognized for its simple implementation, particularly for networks that are already calibrated for the all-or-nothing or the capacity restraint assignment since only minor modifications are needed. Several actual network applications of Burrell's algorithm have been carried out in England and Canada ([14,15,16,17](#)), while the same use of the algorithm has been tried in the United States ([18,19](#))

In theory, Dial's algorithm can be implemented with the capacity-restraint procedure although the algorithms he proposed ([3](#)) are single-pass procedures. Dial's multiple path assignment model has drawn attention from researchers and practitioners of transportation planning since its appearance in 1971. However, the literature focused on the analysis of the algorithm or modification of the algorithm. Very few papers were found that discussed the determination of the parameter θ , calibrations of the model, or guidelines for applications to actual networks. Applications of the model utilize the probabilistic multipath option in the UTPS (UMTA Transportation Planning System) program UROAD ([36](#)).

The literature, as well as unpublished sources, indicate that multiple path assignments produce better results than single minimum path (all-or-nothing) assignments. Some papers ([2,30](#)) even assert that multiple path assignments (including both Burrell's and Dial's algorithms) can achieve convergence of the flow/speed relationship faster than incremental and iterative capacity-restraint assignments. However, these papers do not provide sufficient evidence to support this assertion, nor are there detailed evaluations of the cost effectiveness between multiple path assignments and minimum path assignments.

Strictly speaking, no real "simultaneous-multipath" routing techniques exist in practice. It is obvious that Burrell's path searching mechanisms utilize the all-or-nothing assignment procedure. In Burrell's procedure, the minimum path algorithm is used repeatedly to build minimum path trees and, thus, multiple paths are obtained from different networks built in sequence. Dial's algorithm divides trips among all reasonable routes based on their total route cost compared to the minimum path. The minimum path algorithm is used to provide a basis for the comparisons.

Possibilities for future research concerning the evaluation, development, and improvement of traffic assignment techniques are identified below. The first two items are related to the evaluation of multiple path assignment techniques. The remaining topics are related to the development of new or improved techniques. These topics are:

1. Investigation of the convergence rates: Investigate the convergence rates for Burrell's and Dial's algorithms when they are combined with the capacity-restraint procedure. To carry out this analysis, modifications of existing capacity-restraint assignment models may be needed.
2. Evaluation of the cost effectiveness of various multiple path assignment techniques: Evaluate the techniques by comparing their assignment results, computer run times, data requirements, and other measures of effectiveness. The evaluation should include applications of the techniques to actual networks.
3. The multi-minimum path concept: In a detailed highway network, there may be several paths between a pair of centroids which have similar total impedances. This point is not emphasized by the existing multiple path techniques. The multi-minimum path model should be designed to assign trips to alternative similar cost routes which do not seriously overlap. Such a technique would be especially valuable for modeling congested networks. The literature provides no insight as to how this problem may be solved; hence, it does not appear to be practical at this time.
4. The concept of equalizing volume-to-capacity ratios: The concept is an

extension of the multi-minimum path concept. Each of the similar cost paths should accommodate some of the trips, and the distribution of traffic on these paths should result in V/C ratios which are equal or nearly equal. A procedure based on this concept should more closely reflect actual trip-making characteristics for transportation planning at the project level, such as corridor analysis. A prototype model has demonstrated the concept does produce V/C ratios which are very similar on competing routes. The additional research would include the following: development of an operational version of the model, evaluation of using a realistic application, preparation of user guidelines, and a user's manual.

5. Capacity-restraint assignment techniques attempt to consider alternative paths through a successive adjustment process in which link speeds are adjusted based upon the assigned volume to a coded capacity ratio. However, the existing assignment procedures which apply V/C adjustments to link impedances do not reflect actual conditions. In reality, capacity of a street system is constrained by the arterial-to-arterial intersections. Consequently, the application of the capacity-restraint adjustment to node, instead of link, impedances might be more responsive than the current capacity-restraint procedures. Such a technique has been explained by Hamburg and Associates (37). They developed an intersection analysis model which operates as a submodel within the UTPS highway traffic assignment program UROAD. However, there are many possibilities related to this concept that remain to be investigated.

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APPENDIX A:
LITERATURE REVIEW

LITERATURE REVIEW

A literature search was conducted using key words **TRAFFIC (TRIP) ASSIGNMENT** and the years **1970** through **1990**. Two major data bases in transportation, Transportation Research Information Service (TRIS) and National Technical Information Service (NTIS), were accessed and approximately 800 references were found. By scanning their titles to see if they were related to traffic assignment improvement techniques, 163 references were selected. The conventional traffic assignment techniques include minimum path, capacity restraint, multipath (stochastic), and equilibrium methods. The references were classified in the following eight groups:

- o Advancements of the Conventional Minimum Path Method A-3
- o Advancements of the Conventional Capacity Restraint Method A-5
- o Advancements of the Conventional Multipath Method
(Dial's algorithm) A-8
- o Advancements of the Multiflow Multipath Method
(Burrell's algorithm) A-13
- o Advancements of the Conventional Equilibrium Method A-18
- o Combinations of Various Models A-31
- o Traffic Assignment Techniques at the Operational Level A-38
- o Other Articles Related to the Traffic Assignment Technique A-47

A brief description of each technique is given and is followed by related references. The references and abstracts are taken directly from either TRIS or NTIS and are not the work of this author.

ADVANCEMENTS OF THE CONVENTIONAL MINIMUM PATH METHOD

Traffic assignment is the process of allocating a given number of interzonal trips to an existing or proposed urban transportation network. The traffic assignment problem can be stated as follows:

- Given:
1. A graphic representation of the urban transportation network
 2. The associated link performance (or impedance) functions
 3. An origin-destination matrix (trip table)

Find: The flow (and travel time) on each of the network links.

To solve the traffic assignment problem, the rule by which motorists choose a route must be specified. The algorithms used in various assignment methods define the rules on how motorists choose their routes.

Minimum path assignment (or all-or-nothing) assumes that the link performance for each link of the network is fixed (fixed travel time for each link). Link travel time is not affected by the magnitude of link flow nor is link flow constrained by link capacity. All the trips of a specified origin-destination (O-D) pair will be assigned to the link that yields minimum travel time. The general procedure of the method is described as follows:

1. Obtain the fixed link performance functions (usually link travel time)
2. Trace the minimum path of each O-D pair
3. Assign all the trips of each O-D pair to the minimum path
4. Total the assigned trips on each link of the network

SHORTEST ROUTE ALGORITHM WITH MOVEMENT PROHIBITIONS

Easa, S.M. (Lakehead University, Canada). Transportation Research. Part B: Methodological 19B, No. 3 (June 1985): 197-208, Fig. 4, Tab. 12.

This paper presents an algorithm with movement prohibitions which eliminates some problems encountered in network representation used for traffic assignment models and further allows the representation of the network to be simplified. The paper first presents an appraisal of some proposed methods and reviews the basic concept of existing shortest-route algorithms. The problem of obtaining shortest routes in networks with movement prohibitions is formulated and an algorithm presented. The computational procedure is illustrated by means of a numerical example. The computational efficiency of the algorithm is tested and the test results show that the algorithm is very efficient. (Author/TRRL)

GEOGRAPHIC DECOMPOSITION OF THE SHORTEST PATH PROBLEM, WITH AN APPLICATION TO THE TRAFFIC ASSIGNMENT PROBLEM

Lansdowne, Z.F. and D.W. Robinson (Control Analysis Corporation). Management Science 28, No. 12 (December 1982): 1380-90.

An algorithm, which can be applied to loosely connected networks, is given for geographically decomposing the shortest path problem. The algorithm is applicable to the traffic assignment problem when it is solved as a series of shortest path problems by the Frank-Wolfe algorithm. Numerical results for a large 1287 node, 3752 arc traffic assignment problem for Washington, D.C., indicate that using geographical decomposition can reduce computer memory storage requirements or program run time.

ALL-SHORTEST-TRACKS TRAFFIC ASSIGNMENT

Buultjens, D. (Melbourne University, Australia). Transp Sect Bull N6 (August 1970): 16, Fig. 2, Tab. 2.

This program is designed to perform an all-shortest-paths assignment of interzonal traffic to a street network or a well-connected schematic network. As an option, individual selected-link assignments for up to three given links may be made with or without the full assignment.

ADVANCEMENTS OF THE CONVENTIONAL CAPACITY RESTRAINT METHOD

Conventional capacity-restraint assignments are based either on the iterative or on the incremental procedure. The incremental procedure allocates successive fractions of the total demand to the network on an all-or-nothing basis. Link speeds (travel time) are evaluated and revised on the basis of a predetermined function of volume-to-capacity ratios in each fractional assignment until the total demand is assigned. Best convergence is obtained when decreasing fractions of the total demand are assigned with each successive increment.

The iterative procedure consists of repeated iterations of all-or-nothing assignments of the total demand. Between each iteration, all links are evaluated for their speed-flow (travel time-flow) relationship; and new speeds (travel time) are determined by the next iteration. The resulting network flows for all iterations (usually four) are then averaged to determine the final flow volumes on each link.

In both incremental and iterative assignments, the most widely used link performance function is the Bureau of Public Roads (BPR) formula. The formula adjusts link travel time based on the variation of volume-to-capacity ratios. The equation shown below is a general form of the formula:

$$T_n = T_{n-1} * [1 + \alpha (V/C)^\beta]$$

where:

T_n	=	the new link travel time
T_{n-1}	=	the link travel time of the last iteration
V	=	the assigned volume of that link of the last iteration
C	=	the capacity of that link
α and β	=	the calibrated parameters of that link

In general, both the incremental and iterative assignments are deterministic and heuristic in nature. The main difference between the two procedures is that in the incremental procedure, the results of all previous assignment steps are considered in the evaluation of speeds; while in the iterative procedure, in theory only the results of the immediately preceding iteration are used. However, applications of iterative procedure use averaging or weighted averaging techniques as well.

TRAFFIC ASSIGNMENT BY TRIP TYPE USING VOLUME RESTRAINT AND LINK RESTRAINT FOR APPLICATION IN SMALL URBAN AREAS

Leftwich, D.S., and C.L. Heimbach (University of Central Florida). Journal of Advanced Transportation 18, No. 1 (1984): 55-57.

This research involved the development of a new traffic assignment model consisting of a set of procedures for an urbanized area with a population of 172,000. Historical, social, and economic data were used as input to conventional trip generation and trip distribution models to produce a trip table for network assignment. This fixed table was divided into three trip types: external-external trips, external-internal trips, and internal-internal trips. The methodology used to develop the new traffic assignment model assigned each of the trip types by varying the diversion of trips from the minimum path. A statistical analysis indicated that assigning trips by trip types using trip diversion, volume and link restraint produces a significant improvement in the accuracy of the assigned traffic volumes.

CAPACITY-RESTRAINED ROAD ASSIGNMENT

Vantagnetta, and P.D.C. Dow (Leeds University, England). Traffic Engineering and Control 20, No. 6 (June 1979): 296-305.

The paper is in three sections: (1) the convergence of stochastic methods, (2) the equilibrium methods, and (3) the improved equilibrium method. The convergence of stochastic methods of road assignment such as BURRELL or DIAL are investigated in capacity-restrained networks where the cost of travel on any link depends on the flow of that link. A convergent solution is one where the costs assumed by the route-finding algorithm are identical to those corresponding to the resulting link flows. Both all-or-nothing and DIAL assignments are shown to be inherently non-convergent whereas under certain, quite reasonable, conditions the hypothesis underpinning the BURRELL assignment should lead to convergent solutions. However, simplifications made by BURRELL in the method of the solution for a large network may make the point of convergence extremely difficult, if not impossible, to find. The authors conclude that the instability of these methods, reflected by severe oscillations in flows between iterations, is generally an unavoidable feature of such models. The section on the equilibrium methods reviews the equilibrium methods of assignment to capacity-restraint road networks, methods which are well-known theoretically but have not been used extensively in practice. Their major advantage over heuristic techniques is that they guarantee ultimate convergence to a solution which satisfies Wardrop's first principle of equal and minimum travel costs on all routes used. The principles of equilibrium assignment and a straightforward method based on iterative loading are presented. Results obtained by applying this method to a network of Leeds give better goodness-of-fit statistics with observed counts than conventional methods requiring comparable CPU (Central Processing Unit) times. In addition, the authors find that equilibrium methods are easy to incorporate into existing computer packages, require minimal user intervention, are stable in heavily congested networks and avoid extreme misfits between predicted and observed flows.

Improved equilibrium methods: Two methods of improved solutions to equilibrium assignment models are described. The first, "quantal loading," is in fact based on a technique first used in Chicago over 20 years ago in which updates of the link times or costs are carried out at regular intervals within a single assignment rather than at the end. An extension which applies the same ideas as part of a series of iterative equilibrium assignments is developed. The major advantage of quantal loading is that it greatly accelerates the rate of convergence to Wardrop's equilibrium on a network of Leeds by a factor of 5 on the first iteration and factors of 2 thereafter. The potential reductions in CPU times are, therefore, considerable. The second technique is based on an improved method of combining or averaging different sets of link flows, again with the objective of accelerating convergences and reducing CPU.

Related Article

MULTIPURPOSE TRAFFIC ASSIGNMENT USING VOLUME RESTRAINT AND LINK RESTRAINT FOR APPLICATION IN SMALL URBAN AREAS

Leftwich, D.S., and C.L. Heimbach (University of Central Florida; North Carolina State University, Raleigh). Transportation Research Record N931 (1983): 91-98.

ADVANCEMENTS OF THE CONVENTIONAL MULTIPATH METHOD (DIAL'S ALGORITHM)

Instead of being concerned solely with minimum time paths in a network, the probabilistic multipath method deals with "reasonable" (or efficient) paths. The method assigns trips to all reasonable paths simultaneously in such a way that the resulting effect is identical to what would have been obtained had each path been assigned trips separately under certain choice probability assumptions. The fraction of trips assigned to each alternate link is a diversion probability based on the comparative length and number of reasonable paths through the link.

Two sophisticated algorithms were proposed by Dial in 1971 (R.B. Dial, **A Probabilistic Multipath Assignment Model Which Obviates Path Enumeration**, Transportation Research, Vol. 5, No. 2, 1971).

A simplified procedure of the second algorithm is usually applied in practice (J.L. Edwards and F.O. Robinson, **Multipath Assignment Calibration for the Twin Cities**, Transportation Engineering Journal of ASCE, Vol.103, Nov., 1977). The procedure assigns trips to all reasonable paths between each origin and destination. Each path receives a fraction of interzonal trips which is adversely proportional to path impedance (for example, travel time and/or travel cost.) The notion of the method can be abstracted by the following equation:

$$P = e^{-\theta \delta}$$

where:	P	=	the probability of loading all the trips on a path
	θ	=	diversion parameter; can be specified by users
	δ	=	the difference between minimum path impedance and that of the alternative path

INTERACTIVE GRAPHICS MODEL FOR TESTING STOCHASTIC TRAFFIC ASSIGNMENT ALGORITHMS

Lutin, J.M. (Princeton University). Transportation Research Record N787 (1980): 7-15, Fig. 14.

This paper reports on the use of interactive computer graphics to investigate properties of a stochastic traffic assignment algorithm used to model pedestrian flow in the Urban Mass Transportation Administration's transit station simulation model (USS). The use of an interactive computer language (APL) to transform networks and models into easily manipulable matrix forms is discussed. Graphical calibration of model parameters is discussed and illustrated by using computer plots. The development of an interactive network builder and stochastic traffic assignment model is reported. The exploration of various model forms through interactive graphics is discussed and illustrated by using computer graphics plots of typical networks. (Author)

SELECTED NODE-PAIR ANALYSIS IN DIAL'S ASSIGNMENT ALGORITHM

Van Vliet, D. (Leeds University, England). Transportation Research. Part B: Methodological 15B, No. 1 (February 1981).

An algorithm is proposed to calculate the matrix of trips which use a particular link in a road network or alternatively pass through a particular set of nodes given the route-splitting assumptions inherent in the Dial assignment algorithm. (Author/TRRL)

A STUDY INTO ROUTE CHOICE BY USING A DISAGGREGATE LOGIT MODEL

Hamerslag, R. (Delft University of Technology, the Netherlands). Verkeerskunde 30, No. 8 (August 1979): 377-382.

This study is based on perceived routes used by car drivers. It analyzes the factors which influence route choice. It proved to be impossible to estimate independently the influence of time and distance. Time, however, is an important factor when describing route behavior. Finer specification on links and the use of nodes as explanatory variables in the model result in a better description of the behavior. This approach enables a better link-up with stochastic assignment models. The procedure which is presented here can be used for further development of research into rat-run traffic (fast traffic using secondary roads as by-pass routes) and the consequences of traffic management measures. (TRRL)

MULTIPATH TRAFFIC MODEL WITH DYNAMIC INPUT FLOWS

Robillard, P. (Montreal University, Canada). (March 1973): 21, Fig. 3.

Transportation problems are among the most significant being faced by society today. Much effort is being expended on planning for systems which are efficient, acceptable to man and

compatible with his environment. To solve these problems we need the development of powerful tools that enable the transportation planners to comprehend the phenomena of transportation and to find solutions well proportional to the amount for the stationary case. The assumption made by almost every traffic model in use today in transportation planning is the assumption of the stationarity of the traffic flow where the input flows are assumed constant through time. It is important to refine the presently used traffic models which are supposed to simulate the reality to the case of non-stationary models. The study of the dynamic aspect of the traffic flow patterns, their evolution with time and location, is a key to a better understanding of the transportation phenomena. We present in this study an approach to the realization of this goal by adapting the probabilistic multipath assignment model to the case of non-stationary input flows. The probabilistic multipath assignment model introduced by Dial (TRANS. Res.: 5, pp. 83-111; 1972) is a multipath generalization of the all-or-nothing model and subsumes it as a particular case. Given non-stationary input flows the assignment model evaluates for each point of the transportation network the flow distribution through time. This allows the formation and the evolution of congestions to be studied and gives a more precise image of the reality. The algorithm implementing this dynamic model uses repeatedly an adapted version of the algorithm for the stationary case and a fast Fourier transform procedure.

ON STOCHASTIC MODELS OF TRAFFIC ASSIGNMENT

Daganzo, C.F., and Y. Sheffi. (California University, Berkeley). Transportation Science 11, No. 3 (August 1977): 253-274.

This paper contains a quantitative evaluation of probabilistic traffic assignment models and proposes an alternate formulation. The paper also discusses the weaknesses of existing stochastic-network-loading techniques (with special attention paid to Dial's multipath method) and compares them to the suggested approach. The discussion is supported by several numerical examples on small, contrived networks. The paper concludes with the discussion of two techniques that can be used to approximate the link flows resulting from the proposed model in large networks.

AN EXTENSION OF DIAL'S ALGORITHM UTILIZING A MODEL OF TRIPMAKERS' PERCEPTIONS

Tobin, R.L. (General Motors Corporation). Transportation Research 11, No. 5 (October 1977).

Dial has developed a probabilistic multipath traffic assignment algorithm which is very efficient. However, the assignment of trips to paths is based on a logit model which has some undesirable properties. The purpose of this paper is to extend Dial's algorithm by utilizing a simple model of tripmakers' perceptions and behavior. An important advantage these models have is that they can be calibrated through the study of tripmakers' behavior

on a limited set of trips rather than through the study of origin and destination patterns and link flows on a large network. Three traffic assignment models are presented which are computationally equivalent to Dial's model and which can be implemented by making minor changes in his algorithm. One of the new models, based on path likelihoods contains Dial's model as a special case. The assumptions required about tripmakers' perceptions to obtain this special case seem unreasonable and this explains some of the undesirable properties of Dial's model. In its general form, the new model eliminates some of these undesirable properties. However, like Dial's model, it generally allocates a disproportionate number of trips to portions of a road network in which there is a large number of choices of minor variations in route. The other two models, based on arrival likelihoods and turn likelihoods, both overcome this shortcoming. These two models have different interpretations in terms of tripmaker behavior. The model based on turn likelihoods is the most intuitive and contains Gunnarsson's multipath traffic assignment model as a special case. (Author/TRRL)

MULTIPATH TRAFFIC ASSIGNMENT BASED ON TRIPMAKERS' PERCEPTIONS

Tobin, R.L. (General Motors Research Laboratories; Warren, Michigan). (May 1976): 25 Fig. 3.

General multipath traffic assignment models are presented which are based on a simple model of tripmakers' perceptions and behavior rather than on a heuristic. These models can be calibrated through the study of tripmakers' behavior on a limited set of trips rather than through the study of origin and destination patterns and link flows on a large network. The three models can be implemented by making minor changes in Dial's algorithm. One of the models, based on path likelihoods, contains Dial's model as a special case. The other two new models are based on arrival likelihoods and turn likelihoods.

INVESTIGATION INTO FACTORS AFFECTING THE ROUTE CHOICE IN "RIJNSTREEK-WEST" WITH THE AID OF A DISAGGREGATE LOGIT MODEL

Hamerslag, R. (Delft University of Technology, the Netherlands). Transportation 10, No. 4 (December 1981): 373-391, Fig. 7.

Based on an analysis of observed automobile routes, an investigation is made into the factors affecting choice of routes. As in previous studies, the attempt to determine the influence of time and distance simultaneously was unsuccessful. However, time was found to be an important factor in the description of route choice behavior. An improved description of route choice behavior is achieved if a differentiation according to road type is made or if intersections are included in the model as explanatory variables. The subjective or perceived generalized times caused by intersections are approximately 0.88 minutes, and those caused by left-turn movements are 2.55 minutes. The result could further be improved by dividing the difference in generalized times between two routes by the root of the generalized times. A great advantage of this approach is also that it links up better with stochastic assignment models in which traffic is randomly distributed over

alternative routes. In particular for studies on extraneous traffic and the consequences of traffic-controls, the method presented here serves as a start for further investigation. (Author/TRRL)

A MODEL AND ALGORITHM FOR MULTICRITERIA ROUTE-MODE CHOICE

Dial, R.B. (U.S. Department of Transportation). Transportation Research. Part B: Methodological 13B, No. 4 (Dec 1979): 311-316.

Using an idea proposed independently by Quandt and Schneider, the paper declares mode choice to be a special instance of route choice. It describes a (non-logit) model which includes in its route-choice mechanism the decision variables traditionally associated with mode choice. With the assumption that each traveller selects the route which minimizes his own personal linear choice function, it is clear that the routes with a nonzero chance of being picked are only those not dominated by any other path (e.g., are not both costlier and longer than any other path). The precise probability of a route being chosen is just the integral over the appropriate portion of the probability density of the coefficients of the choice function. The integration limits are implied by the amount of each disutility on each of the undominated routes. An algorithm is given which is quite efficient in finding these paths in a large and complex multi-modal network.

Related Article

UNDERSTANDING DRIVER ROUTE CHOICE

Southern, A.C. Planning and Transport Research and Computation (1988): 13-26.

ADVANCEMENTS OF THE MULTIFLOW MULTIPATH METHOD

The multiflow multiple routing method is based on the assumption that routes vary because drivers make different estimations of the behavioral cost which they would incur by using the links making up their possible routes. The basic concept of this method was first developed by the primary author in association with Burrell in London, England, in 1968 (J.E. Burrell, **Multiple Route Assignment and Its Application to Capacity Restraint**, Freeman, Fox, Wilbur Smith and Associates, London. Paper presented to the 4th International Symposium on the Theory of Traffic Flow, Karlsruhe, Germany, 1968).

Burrell proposed an assignment algorithm that builds multiple routes between nodes rather than zones (load nodes). The process assumes that people do not have an exact knowledge of the time required for a trip and use instead a random distribution of link times about the "true" link time as the basis for minimum-path routing. All the travelers from one zone are assumed to have the same perception of link travel times (to reduce computational complexity) where these travel times are drawn at random from a normal distribution with the mean of the "true" time and a certain percent of the true time as standard deviation. A set of minimum-time paths is then obtained for that zone to all other zones.

A possible distribution of costs is, therefore, associated with each link, and the probability of a particular route being selected is obtained by combining the link cost distributions to give the probability of the route in question having the lowest estimated cost. The values specified in the distributions are proportionate deviations from the link length (conceptual length = travel time) specified in the network. Each link obtains its random values by varying the mean deviation according to the link length and its class. For instance, the link length distribution applied to a link of length C may have a number of variations of the form:

$$C' = C (1 + y_i k / \sqrt{C})$$

where:

C'	=	the random value of link length
C	=	the "true" link length (mean length)
y_i	=	a probability distribution with zero mean and unit mean deviation
k	=	the mean deviation applied to a link of unit length

The user controls the amount of multiple routing by specifying k/\sqrt{C} which is the ratio of the mean deviation of each link length distribution to the coded link length.

CALIBRATION OF THE STOCHASTIC MULTIPLE-ROUTE ASSIGNMENT MODEL - TRANSPORTATION PLANNING METHODS

Gommers, M.J.P., and P.H.L. Bovy. Seminar M (Sussex University, England). PTRC Education and Research Services, Ltd. Report No. P282 (1986): 137-50.

The stochastic assignment model (an example of which is the Burrell model) is one of the assignment techniques enabling multiple route choice. In contrast to the all-or-nothing and equilibrium (capacity-restraint) techniques, it is not widely used. Despite its theoretical appeal the stochastic assignment model yields in general no better empirical results than the simple all-or-nothing model. Apart from the applications of this model to the wrong situations (with congestion), a major reason for this is presumably an inadequate calibration of the parameter(s). So far, little attention has been paid to a practical calibration technique. This paper will deal with a calibration method using simulation techniques. The paper first describes the stochastic assignment model as a multinomial probit model within the framework of random utility discrete choice theory. Maximum likelihood theory offers the basis for deriving an approximate calibration statistic for use with link counts which, however, can only be estimated through repeated simulations. The paper discusses statistical requirements for the calibration procedure and proposes a strategy with the added advantage of a useful interpretation of the route choice situation at hand. Attention will be paid to various aspects of the simulation process (random numbers, number of simulation runs and variance-reduction techniques). The suggested procedure has been used for calibrating a bicycle route choice and assignment model for the city of Delft, the Netherlands. The calibration is based on observations of routes. The calibration results are very reasonable and demonstrate the outcomes of a simple but well-calibrated stochastic assignment model to be a clear improvement compared to those of an all-or-nothing model. For the covering abstract of the seminar see IRRD 812674. (Author)

CALIBRATION CHARACTERISTICS OF THE STOCHASTIC MULTIPLE ROUTE ASSIGNMENT MODEL

Gommers, M.J.P., and P.H.L. Bovy (Traffigo, Rotterdam; Delft University of Technology, the Netherlands). Colloquium Vervoersplanologisch Speurwerk. (1985): 179-204, Fig. 7, Tab. 2.

The stochastic assignment model is one of the assignment models enabling multiple route choice. In contrast to the all-or-nothing and equilibrium (capacity restraint) techniques, it is not very widely used, possibly due to its poor results relative to the much simpler all-or-nothing model. One of the reasons for this unsatisfactory performance might be the insufficient calibration of the unknown parameters. This paper deals with the calibration problem of stochastic assignment models. Attention is given to the goodness-of-fit measure, the form of the calibration curve, as well as to technical aspects of the simulation process such as number of trials, random number generation, etc. Empirical results are reported referring to the calibration of the model for bicycle trips in the city of Delft. For the covering abstract of the conference see TRIS 453956. (Author/TRRL)

HIGHWAY ASSIGNMENT MULTIFLOW: A CRITICAL APPRAISAL

Gilliver, I. (Reading University, England). White Knights Reading RG6 (October 1977): 62, Fig. 11, Tab. 9.

The report reviews approaches to highway assignment in the context of the transportation planning process and the general theory of assignment. The multiflow model is examined in detail, with simulations run to test model properties and to compare the relative performance of alternative distributions of perceived link costs. The possibility of developing an analytical approach is discussed. The model is applied to a real network and the stability of assignment is examined under random perturbation of perceived costs. The work is related to the debate on generalized cost/time-only approaches to assignment and its consistency with the conventional evaluation methodology discussed. (TRRL)

A CONVERGENT PROBABILISTIC ROAD ASSIGNMENT MODEL

Randle, J. (Greater Manchester Council, England). Traffic Engineering and Control 20, No. 11 (November 1979): 519-521, Fig. 6, Tab. 1.

The paper describes a model combining non-minimum cost routes with a proper treatment of capacity restraint. The route choice model using the Burrell behavioral theory is described followed by the treatment of capacity restraint and convergence. Examples are given of the use of Van Vliet's notation to determine cost probabilities. An application of the model for a large urban network is presented. The advantages of the model are discussed. The model is shown to be perfectly converged for a large, congested urban network. Resultant link costs and volumes simultaneously satisfy a route choice model and a set of cost flow curves. Three iterations are normally sufficient. Link profiles are produced and used in a demand capacity-restraint assignment. "Faced convergence" produces the final link cost and consequently the final link volume. In addition, the model can be combined with methods of secondary analysis on the converged network to be completely compatible with the final assigned link. (TRRL)

NEW APPROACH TO TRANSPORTATION SYSTEMS MODELING

Low, D.E. Traffic Quart Eno Fdn for Transp 26, No. 3 (July 1972): 391-404.

In this model traffic volumes are determined one link at a time, primarily as a function of the relative probability that one link will be used in preference to another. Interzonal trip probabilities are then assigned to the network using regular traffic assignment procedures to produce estimated trip probabilities on a link-by-link basis. Regression equations are then developed to relate counted link volumes to assigned trip probabilities and other link characteristics. These equations can then be used to estimate link volumes under other conditions after determining and assigning new interzonal trip probabilities to reflect those conditions. This approach can be used to produce traffic volume forecasts when study funds or input data required for the usual techniques are lacking. (Author)

USE OF A MULTIPLE ROUTING TECHNIQUE FOR ONE PASS CAPACITY RESTRAINT ASSIGNMENTS

Wildermuth, B.R. Traffic Quart Eno Fdn Hwy Traffic Control 26, No. 2 (April 1972): 247-65.

Multiple routing, as presented here, adds through its stochastic principle a long-sought element of realism to the traffic assignment process. This principle eliminates the problem associated with the minimum path procedure, which leads to extremely unbalanced flows where more than one acceptable route exists between pairs of nodes. With the multiple routing principle as its basic framework, a one-pass incremental capacity-restraint procedure is presented which provides a flexible and economical tool for producing traffic assignments with stable speed-flow relationships. Results obtained with this procedure indicate a level of assignment accuracy which is equivalent or superior to that obtained with four iterations of the frequently used averaging procedure. (Author)

DRIVERS' ROUTE CHOICE PROJECT PILOT STUDY

Carpenter, S.M. (Oxford University, England). Transport Studies Unit. Report No. TSU/REF-085/WP42 (March 1979): 53.

Current practice during the design of traffic assignment models is to assume that drivers' choice of route in a given network will be optimal, based on the minimization of either journey time, travel distance or generalized cost. The aim of this survey is to establish the factors which drivers take into account when choosing a route, as in practice, drivers' decisions may be affected by various factors such as the journey purpose, their familiarity with the area, the length of the journey and their use of road maps and signposts.

THE CHOICE OF ROUTE, MODE, ORIGIN AND DESTINATION BY CALCULATION AND SIMULATION

Robertson, D.I., and Janet V. Kennedy (Crowthorne, England). Transport and Road Research Lab. Report No. TRRL-LR-877 (1979): 34.

People have to make choices when they make trips. The choices may be of destination, of mode of travel, of route or of time. The report describes a simple method of calculating the assignment of trips where travelers are free to choose between alternatives. The method of calculation is called 'SCATA' (Simple Choice Algorithm for Trip Assignment) and is derived from the assumption that travelers vary in their perception of the costs and benefits of the alternatives and choose whichever is best for them. The use of SCATA is illustrated by examples that cover choice of route, of mode and of origin and destination. The theoretical accuracy of SCATA is assessed by comparing the assignments with those predicted by a simulation that is based upon the same fundamental assumptions. It is thought that SCATA may be of use in the design of traffic management schemes. For this

purpose, the SCATA method of assignment must be joined to a model that predicts trip costs as a function of the assignment. An example is given that shows some of the implications that follow when SCATA is used and the implications that follow when SCATA is used in this way.

Related Article

THE BEHAVIORAL DYNAMICS OF ROUTE CHOICE

Charlesworth, J.A., and E. Gunawan. Traffic Engineering and Control 28, No. 2 (February 1987): 80-83.

ADVANCEMENTS OF THE CONVENTIONAL EQUILIBRIUM METHOD

The basic concept of equilibrium traffic flow in a network system is that no traveler can reduce his or her travel time by switching to other routes if the system is under equilibrium condition. Two alternative criteria used to determine the traffic allocation were proposed by Wardrop in 1952 (J. G. Wardrop, **Some Theoretical Aspects of Road Traffic Research**, Proc., Institution of Civil Engineers, Part 2, Vol.1, 1952): (1) the journey times on all the routes actually used are equal and (2) the average journey time is minimum. In short, the equilibrium of traffic flow is achieved by two approaches: (1) user optimization and (2) system optimization.

One exemplary method proposed by Eash, Janson, and Boyce in 1979 (R. W. Eash, B. N. Janson, and D. E. Boyce, **Equilibrium Trip Assignment: Advantages and Implications for Practice**, Transportation Research Record, No. 728, Washington, D. C., 1979) recommended that the equilibrium method be used in large, congested urban transportation networks according to the principle of equal travel time. The equilibrium assignment of trips can be found by solving a set of nonlinear programming problems, minimizing the total vehicle-travel time subject to user equilibrium conditions and flow conservation equations. The procedure of the proposed method is summarized as follows:

1. Compute the travel time ($S_a(V_a)$) on each link that corresponds to the flow (V_a) in the current solution (usually the BPR equation is used to compute the link travel time).
2. Trace the minimum path of each O-D pair using the link travel times from Step 1.
3. Assign all the given O-D demand using the all-or-nothing assignment procedure; the new assigned volume is signed as W_a .
4. Combine the current solution assignment (V_a) and the assignment from Step 3 (W_a) to obtain a new current solution (V_a') by using a value α selected so as to minimize the following objective function:

$$\sum_a \int_0^{V_a'} S_a(x) dx$$

$$\text{where } V_a' = (1-\alpha)V_a + \alpha W_a$$

- After V_a' is obtained, go to Step 1.
5. Repeat Steps 1 through 4 for several iterations until the solution has converged sufficiently. This indicates that the objective function is essentially minimized and the travel times on different paths for each O-D pair are almost equal.

THE EQUIVALENCE OF TRANSFER AND GENERALIZED BENDERS DECOMPOSITION METHODS FOR TRAFFIC ASSIGNMENT

Barton, R.R., D.W. Hearn, and S. Lawphongpanich. Transportation Research. Part B: Methodological 23B, No. 1 (February 1989): 61-73, Fig. 6.

In prior work we have given an intuitive development of Transfer Decomposition, a decomposition of the traffic assignment problem into two traffic assignment problems. The intent of this paper is to provide a rigorous basis for this technique by establishing that it is a generalized Benders decomposition. As an illustration of the result, we give a decomposition algorithm that is based on the familiar Frank-Wolfe method.

AN EXAMINATION OF CONVERGENCE ERROR IN EQUILIBRIUM TRAFFIC ASSIGNMENT MODELS

Rose, G., M.S. Daskin, and F.S. Koppelman. Transportation Research. Part B: Methodological 22B, No. 4 (August 1988): 261-74.

This research focuses on the convergence error inherent in the predictions of network equilibrium flows obtained from the Frank-Wolfe algorithm. Factors which determine the magnitude of the convergence error are identified. The effects of selection of a stopping criterion and an initial solution are examined in an empirical study. The results indicate that it is difficult to obtain precise flow predictions in some contexts, specifically where networks contain only a few O-D pairs which are connected by a number of alternative overlapping routes. In addition to having important implications for research studies, the empirical results highlight the need for both researchers and practitioners to consider the issue of convergence error when selecting stopping criteria and initial solutions for flow prediction algorithms. (Author)

COMPARISON OF SOME ALGORITHMS FOR EQUILIBRIUM TRAFFIC ASSIGNMENT WITH FIXED DEMANDS - TRANSPORTATION PLANNING METHODS

Arezki, Y. Seminar M (Sussex University, England). PTRC Education and Research Services, Ltd. Report No. P282 (1986): 123-35.

In this report we show how to improve the Frank-Wolfe algorithm solving the equilibrium traffic assignment problem with fixed demand by applying (1) the partan technique, (2) a heuristic partan, (3) an accelerated method, (4) a method due to Fukushima, and (5) quantal loading. All algorithms were coded in FORTRAN and run on an Amdhal at the University of Leeds. Tests were carried out for three real networks in order to assess the differences between the techniques with respect to CPU time and number of iterations for various sized networks. The smallest network consisted of 24 zones, 209 nodes and 321 links; the largest consisted of 148 zones, 932 nodes and 174 links. The heuristic partan was the most successful algorithm for most networks. The same level of convergence after 30 iterations of the standard Frank-Wolfe could be obtained after 10 iterations with the

heuristic partan. Quantal loading gave very satisfactory results for networks with very steep cost flow curves particularly over the early iterations. The accelerated method performed well for most networks but needs to be empirically calibrated for individual networks. The Fukushima method was successful for some networks but was not consistently reliable. For the covering abstract of the seminar see IRRD 812674. (Author)

EQUILIBRIUM TRAFFIC ASSIGNMENT WITH MULTIPLE USER CLASSES - TRANSPORTATION PLANNING METHODS.

Van Vliet, D., T. Bergman, and W.H. Scheltes. Seminar M (Sussex University, England). PTRC Education and Research Services, Ltd. Report No. P282 (1986): 111-22.

This paper considers the problems of dividing the trip matrix to be assigned to a road network into "multiple user classes" where the division of trips is based on (a) physical characteristics of the vehicles, (b) driver characteristics, and/or (c) network restrictions. A Wardrop equilibrium model is formulated to deal with conditions where travel costs are a function of flows. Under certain, not unrealistic, conditions this model is shown to be equivalent to a convex minimization problem, and a convergent assignment procedure based on the Frank-Wolfe algorithm is proposed. This algorithm has been implemented and tested using the SATURN suite of programs. Practical considerations in applying the model to networks in the Netherlands are discussed. For the covering abstract of the seminar see IRRD 812674. (Author)

ACCELERATING CONVERGENCE OF THE FRANK-WOLFE ALGORITHM

Weintraub, A., and C. Ortiz (Chile University, Santiago). Transportation Research. Part B: Methodological 19, No. 2 (April 1985).

We consider the Frank-Wolfe algorithm in the context of the traffic assignment problem. The slow-convergence characteristics close to the optimum solution of this popular approach are well known. Several proposals have improved on the original method by modifying the search direction. We propose modifying the step size, which leads to very significant improvements in efficiency. (Author/TRRL)

CONVEX PROGRAMMING FORMULATIONS OF THE ASYMMETRIC TRAFFIC ASSIGNMENT PROBLEM

Hearn, D.W., S. Lawphongpanich, and S. Nguyen (Florida University, Gainesville; Montreal University, Canada). Transportation Research. Part B: Methodological 18B, No. 4/5 (1984).

Recently introduced optimization formulations of the asymmetric traffic assignment problem are developed. The duality of two formulations is shown, and conditions for convexity and differentiability of the objective functions are given. Convexity conditions are also given for the family of formulations recently introduced by Smith (1983). When the travel cost vector

is affine and monotone, all of the formulations are shown to be convex programming problems. Algorithmic implications of the results are discussed. (Author/TRRL)

A COMPARISON OF USER-OPTIMUM VERSUS SYSTEM-OPTIMUM TRAFFIC ASSIGNMENT IN TRANSPORTATION NETWORK DESIGN

LeBlanc, L.J., and M. Abdulaal (Vanderbilt University, Nashville, TN; Alfateh University, Tripoli, Libya). Transportation Research. Part B: Methodological 18B, No. 2 (April 1984).

In this report, we compare the computational efficiency and results of solving two alternative models for the problem of determining improvements to an urban road network. Using a 1462 link, 584 node test network of the North Dallas area, we compare a model which assumes user-optimum behavior of travelers with a model which assumes system-optimum flows. Both of these models allow improvements to the road network to take on any non-negative value, rather than requiring discrete improvement values. Investment costs are modelled by functions with decreasing marginal costs. Unfortunately, the user-optimum model, which is much more realistic than the system-optimum one, normally cannot be solved optimally. However, the simpler system-optimum model can be optimally solved, provided that investment costs are approximated by linear functions. Thus, for this network design problem we compare an accurate representation which can be solved only approximately with an approximate representation which can be solved optimally. Our computational testing showed that the system-optimum model produces solutions as good as those from the user-optimum model, and thus seems justified when favored by other considerations, such as ease of coding, availability of "canned" programs, etc. (Author/TRRL)

EQUILIBRIUM TRAFFIC ASSIGNMENT ON AN AGGREGATED HIGHWAY NETWORK FOR SKETCH PLANNING

Eash, R.W., K.S. Chon, Y.J. Lee, and D.E. Boyce. Transportation Research Record N944 (1983): 30-37, Fig. 6, Tab. 5.

An application of the equilibrium traffic assignment algorithm on a simplified highway network, such as might be used for sketch planning is described. Analysis zones in the assignment are also substantially larger than in most conventional traffic assignments. The algorithm for equilibrium traffic assignment is introduced, followed by a discussion of the problems with equilibrium traffic assignment in a sketch-planning application. Next, the network coding procedures for the case study are examined. Results of the sketch-planning assignment are then evaluated against a comparable regional assignment of the same trips. Finally, there is a discussion of how this research fits into the programs of a transportation planning agency. (Author)

A MODIFIED FRANK-WOLFE ALGORITHM FOR SOLVING THE TRAFFIC ASSIGNMENT PROBLEM

Fukushima, M. (Kyoto University, Japan). Ion Centre, Cowper Works. Bumpus, Haldane and Maxwell, Ltd. 18B, No. 2 (April 1984): 169-177, Tab. 2.

This paper presents a very simple modification of the Frank-Wolfe algorithm for the solution of the traffic assignment problem. It is shown that the modified algorithm can be implemented without much increase in computational effort over the original one. Convergence of the algorithm is proved and computational results are reported to demonstrate the validity of the modification. (Author/TRRL)

TWO ALTERNATIVE DEFINITIONS OF TRAFFIC EQUILIBRIUM

Smith, M.J. (York University, England). Transportation Research. Part B: Methodological 18B, No. 1 (February 1984): 63-65.

The paper shows that, under reasonable conditions, if there are junction or modal interactions, a Wardrop equilibrium exists, but a user-optimized flow may not. (Author/TRRL)

AN ALGORITHM FOR SOLVING ASYMMETRIC EQUILIBRIUM PROBLEMS WITH A CONTINUOUS COST-FLOW FUNCTION

Smith, M.J. (York University, England). Transportation Research. Part B: Methodological 17B, No. 5 (October 1983): 365-371.

This paper gives two user objective functions for the asymmetric assignment problem, and an algorithm of descent type. The algorithm produces a sequence of flows which converges to the set of equilibria if the cost-flow function is continuous. (Author/TRRL)

MULTICRITERIA SPATIAL PRICE EQUILIBRIUM NETWORK DESIGN: THEORY AND COMPUTATIONAL RESULTS

Friesz, T.L., and P.T. Harker (Pennsylvania University, Philadelphia). Transportation Research. Part B: Methodological 17B, No. 5 (October 1983): 411-426.

In this paper we consider the problem of determining the optimal design of a transportation network using a vector valued criterion function when the flow pattern is assumed to correspond to a spatial price equilibrium. This problem arises in rail and freight network design where the spatial price equilibrium is a better behavioral description than the Wardropian user equilibrium characteristic of urban transportation applications. We describe two alternative heuristic solution techniques for the multicriteria spatial price equilibrium network design problem. The first is based on iteration between a pure spatial price equilibrium model and a vector optimization model with only non-negativity

constraints. The second solution technique employs the Hooke and Jeeves algorithm for nonlinear programming to solve a vector optimization model with implicit constraints guaranteeing a spatial price equilibrium flow pattern. In these solution procedures, rather than represent the equilibrium problem as a mathematical program, as is normally done for the Wardropian traffic assignment problems used in urban applications, we employ an original nonlinear complementarity formulation of the spatial price equilibrium problem written entirely in terms of nodal and arc variables and solved extremely efficiently through the iterative application of a linear complementarity algorithm. The nonlinear complementarity formulation allows us to address problems with asymmetric transportation cost, commodity demand and commodity supply functions without the specialized diagonalization/relaxation algorithms required by other approaches. (Author/TRRL)

NETWORK AGGREGATION EFFECTS UPON EQUILIBRIUM ASSIGNMENT OUTCOMES: AN EMPIRICAL INVESTIGATION

Bovy, P.H., and G.R.M. Jansen (Delft University of Technology, the Netherlands). Operations Research Society of America. Transportation Science 17, No. 3 (August 1983): 240-262.

Three network models were developed for the road network of Eindhoven (population = 200,000): a fine, a medium and a coarse network model. In this article the results of the equilibrium assignments are presented which are occasionally compared with all-or-nothing outcomes. Many load figures are dealt with here. The experiment indeed showed a significant effect of the level of detail on most assignment outcomes. This effect proved to be consistent but diminishing; an increase in the level of detail always yielded better results but only marginal improvement could be obtained beyond a certain level. Compared with all-or-nothing assignment results, equilibrium loads agree much better with the counts at all levels of spatial detail.

THE EXISTENCE AND CALCULATION OF TRAFFIC EQUILIBRIA

Smith, M.J. (York University, Canada). Transportation Research. Part B: Methodological 17B, No. 4 (August 1983): 291-303.

This paper considers the assignment problem when there are junction interactions. We give an objective function which measures the extent to which a traffic distribution departs from equilibrium, and an algorithm which (under certain conditions) calculates equilibria by steadily reducing the objective function to zero. It is shown that the algorithm certainly works if the network cost-flow function is monotone and continuously differentiable and a boundary condition is satisfied. (Author/TRRL)

AN EQUILIBRIUM ALGORITHM FOR THE SPATIAL AGGREGATION PROBLEM OF TRAFFIC ASSIGNMENT

Daganzo, C.F. Transportation Research. Part B: Methodological 14B, No. 3 (September 1980): 221-28.

Present traffic assignment methods require that all possible origins and destinations of trips within a study area be represented as if they were taking place to and from a small set of points or centroids, each of which represents the location of all trip ends within a given zone. Since this necessarily misrepresents points located at the zone edges, smaller zones and more centroids could be used, but existing traffic assignment algorithms cannot efficiently handle many centroids. An algorithmic procedure has been developed to provide more than one centroid per zone. These multiple centroids or "subcentroids" are connected to the network at selected "access nodes" (either in or out of the zone), but the permissible number of centroids is still limited. See also HS-030 090.

CONGESTION SENSITIVE ASSIGNMENT - THE APPLICATION OF THE EQUILIBRIUM-ASSIGNMENT MODEL

Jansen, G.R.M. (Delft University of Technology, the Netherlands). Royal Dutch Touring Club ANWB. Verkeerskunde 33, No. 4/5 (April 1982): 236.

Assignment models often show a disadvantage: while the assignment process loads the network with traffic, the results do not show the way in which the remainder of the journeys are routed. This may lead to unrealistic results. This article deals with an equilibrium assignment model which is sensitive to congestion and therefore leads to more balanced network loadings. (TRRL)

CONVERGENCE OF THE FRANK-WOLFE METHOD FOR CERTAIN BOUNDED VARIABLE TRAFFIC ASSIGNMENT PROBLEMS

Hearn, D.W., and J. Ribera (Florida University, Gainesville). Transportation Research. Part B: Methodological 15B, No. 6 (December 1981): 437-442, Fig. 1, Tab. 2.

In two recent papers published in transportation research, Daganzo presented a modification of the Frank-Wolfe algorithm to solve certain link capacitated traffic assignment problems satisfying certain conditions. In order to show convergence of the modified algorithm, the assumption was made that the integral of the volume delay formula for each link tends to infinity as the link flow approaches the link capacity. In this paper we give a theorem which establishes convergence of the modified algorithm under much weaker conditions. This result is then used to show convergence if the objective function of the assignment model is sufficiently large (not necessarily infinite) when the link flows are at capacity. Thus the modified method is applicable to a broader class of assignment problems. Two numerical examples illustrate (a) when the method converges and when it does not, and (b) that our theorem provides a weaker condition for convergence of the method. (Author/TRRL)

THE EXISTENCE OF AN EQUILIBRIUM SOLUTION TO THE TRAFFIC ASSIGNMENT

PROBLEM WHEN THERE ARE JUNCTION INTERACTIONS

Smith, M.J. (York University, Canada). Transportation Research. Part B: Methodological 15B, No. 6 (December 1981): 443-451.

We consider a network with interactions and capacity constraints at each junction. We give conditions on the interactions and constraints which, if satisfied at each separate junction, ensure that any feasible assignment problem has an equilibrium solution. Two illustrative examples are provided; the first arises naturally and does not satisfy our conditions, while the second does satisfy our conditions but is somewhat unnatural. (Author/TRRL)

EMPIRICAL EVIDENCE FOR EQUILIBRIUM PARADOXES WITH IMPLICATIONS FOR OPTIMAL PLANNING STRATEGIES

Fisk, C, and S. Pallottino (Montreal University, Canada; Istituto Applicazioni Calcolo, Rome). Transportation Research. Part A: General 15A, No. 3 (May 1981).

In a recent paper it was shown using a simple network model that the total travel cost in a user equilibrium assignment may decrease if some input flows are increased. It is shown here using the city of Winnipeg network data that this phenomenon may occur in "real life." The significance of these results for the problem of determining optimal planning strategies and network modelling are discussed. (Author/TRRL)

NETWORK EQUILIBRATION WITH ELASTIC DEMANDS

Gartner, N.H. (Lowell University). Transportation Research Record No. 775 (1980): 56-61, Fig. 5.

Elastic-demand equilibration (assignment) is an analytical model for travel forecasting in homogeneous and multimodal transportation networks in which the demand for travel between each origin-destination (O-D) pair is an elastic function of the service level offered by the network. The problem was formulated as a mathematical optimization program in 1956, and since that time, a variety of iterative schemes have been proposed for its solution. In this paper, the mathematical-programming formulation of the network-assignment problem (NAP) with elastic demands is examined, an economic rationale for its optimization objective is derived, and an efficient method for its solution is presented. The method is based on modelling the NAP as an equivalent-assignment problem in an expanded network. The variable-demand NAP is thus transformed into a fixed-demand NAP that has a trip table that consists of the potential O-D travel demands and can therefore be solved by any fixed-demand assignment procedure available.

ARRBTRAFIC: PROGRAM USER MANUAL - EQUILIBRIUM TRAFFIC ASSIGNMENT FOR FIXED DEMANDS

Luk, J.Y.K., A. Makarov, and M.R. Wigan. Australian Road Research Board. (October 1979): 37.

This manual is a revised edition of ATM No. 4 (IRRD No. 229983). The report provides detailed documentation and examples of input and output for a family of four equilibrium traffic assignment programs. ARRBTRAFIC/CDC CYBER (1) is useful for an accurate prediction of traffic flows according to the first Wardrop equilibrium assignment principle (user cost minimisation). ARRB/TRANPLAN includes subroutines to interface with other travel demand programs to form a comprehensive planning system for generation, distribution and assignment. ARRBTRAFIC/NOVA-840 is suited to educational and research purposes and offers more options than the other versions. ARRBTRAFIC/CDC-CYBER (2) runs considerably faster and uses the recent Dial-Klingman-Glover shortest path algorithm. An optional direct calculation method is included for combining two consecutive solutions, but may produce less accurate results when this option is selected. A generalized piecewise speed-flow relationship is also provided to permit users to specify their own speed-flow relationship for a particular assignment study. The delay function provided by the user may be weighted by up to fifteen different values of capacity corresponding to differing link types. It also offers the option of sorting two-way links in the output listing. A microfiche listing of the four different versions of the code is included at the rear of this manual. (Author/TRRL)

MORE PARADOXES IN THE EQUILIBRIUM ASSIGNMENT PROBLEM

Fisk, C. (Montreal University, Canada). Transportation Research. Part B: Methodological 13B, No. 4 (December 1979): 305-309, Fig. 3.

The sensitivity of travel costs to changes in input flows in the Wardrop equilibrium problem is studied. Examples are given showing that both origin to destination and global travel costs may decrease as a result of an increase in input flows. Other examples show that in the two-mode equilibrium assignment problem, transit origin to destination travel costs may decrease as a result of an increase in automobile input flows. (Author/TRRL)

MAXIMUM ENTROPY COMBINED DISTRIBUTION AND ASSIGNMENT MODEL SOLVED BY GENERALIZED BENDER'S DECOMPOSITION

Joernsten, K.O. (Linköping University, Sweden). *Matematiska Institutionen*. S-58183 (1975): 5, Fig. 5, Tab. 3.

The combined distribution and assignment problem has been studied by many authors. The problems examined have been of different types. Some authors focus their attention on the user optimized combined distribution and assignment problem whereas others study a system optimized version. Apart from this difference there is also a difference in the way the distribution part and the assignment part are incorporated into the models. In this paper is studied a combined distribution and assignment problem with entropy objective and cost

constraints, and an algorithm is given based on generalized Bender's decomposition for its solution. The resulting solution method will be a "dual" method. (TRRL)

SOME DEVELOPMENTS IN EQUILIBRIUM TRAFFIC ASSIGNMENT

Fisk, C. (Montreal University, Canada). (January 1979): 26, Fig. 2, Tab. 6.

A network optimization problem is formulated which yields a probabilistic equilibrated traffic assignment incorporating congestion effects and which, as a special case, reduces to a user optimized equilibrium solution. The article also demonstrates the similarity between some fixed demand incremental methods of traffic assignment and the minimization problem associated with computing the user equilibrium assignment. (TRRL)

CONTINUOUS EQUILIBRIUM NETWORK DESIGN MODELS

Abdulaal, M., and L.J. LeBlanc (Southern Methodist University). Transportation Research. Part B: Methodological 13B, No. 1 (March 1979): 19-32, Fig. 3, Tab. 4.

It is known that the road network design problem with the assumption of user optimal flows can be modelled as a 0-1 mixed integer programming problem. Instead, we formulate the network design problem with continuous investment variables subject to equilibrium assignment as a nonlinear optimization problem. We show that this optimization problem is equivalent to an unconstrained problem which we solve by direct search techniques. For convex investment cost functions, the performance of both Powell's method and the method of Hooke and Jeeves is approximately the same with respect to computational requirements for a 24 node, 76 arc network. For the case of concave investment functions, Hooke and Jeeves are superior. The solution to the concave continuous model is very similar to that of the 0-1 model. Furthermore, the required solution time is far less than that required by the corresponding discrete model of the same network. The advantages and disadvantages of the continuous approach as well as the computational requirements are discussed. (TRRL)

FORMULATING AND SOLVING THE NETWORK DESIGN PROBLEM BY DECOMPOSITION

Dantzig, G.B., R.P. Harvey, Z.F. Lansdowne, D.W. Robinson, and S.F. Maier (Stanford University; Control Analysis Corporation; Duke University). Transportation Research. Part B: Methodological 13B, No. 1 (March 1979): 5-17, Fig. 1.

The optimal transportation network design problem is formulated as a convex nonlinear programming problem, and a solution method based on standard traffic assignment algorithms is presented. The technique can deal with network improvements which introduce new links, increase the capacity of existing links or decrease the free-flow (uncongested) travel time on existing links (with or without simultaneously increasing link

capacity). Preliminary computational experience with the method demonstrates that it is capable of solving very large problems with reasonable amounts of computer time. (Author/TRRL)

IMPROVED LINEAR APPROXIMATION ALGORITHM FOR THE NETWORK EQUILIBRIUM (PACKET SWITCHING) PROBLEM

Florian, M (Montreal University, Canada). The 16th Institute of Electrical and Electronics Engineers Conference Decision Control. (1977): 812-818.

A method is presented for accelerating the convergence of the algorithm based on the Frank-Wolfe linear approximation method for the convex cost multicommodity flow problem known as the "equilibrium traffic assignment problem" in transportation networks and as the "optimal routing of packet switched messages" in communication networks. The acceleration of the convergence of this algorithm is achieved with a nontrivial adaptation of Wolfe's suggestion of an "away" step in the linear approximation method and a variant of this adaptation based on restriction.

ON THE TRAFFIC ASSIGNMENT PROBLEM WITH FLOW DEPENDENT COSTS - I AND II

Daganzo, C.F. (Massachusetts Institute of Technology). Transportation Research 11, No. 6 (1977): 433-439, Fig. 4.

The author refers to a paper by LeBlanc, et al., (1975) which presented a very efficient mathematical algorithm method to solve the multicommodity user-equilibrated network flow distribution problem with flow-dependent costs. This method is considered to be similar in computational requirements to the iterated capacity restrained technique but to be mathematically proven to converge. This paper shows how to generalize the algorithm to networks with link capacities, and a simple modification to LeBlanc's original algorithm has been introduced which makes possible the study of capacitated networks. The modification is considered to be mathematically sound and not to add any significant computational burden to the algorithm. Its application is illustrated by an example in which LeBlanc's notation has been used. In the second part of this paper the previous results are generalized by showing how to incorporate link capacities into any equilibrium traffic assignment algorithm. This is done by defining an equivalent uncapacitated traffic equilibrium problem (ep). As an example, the performance of the convex-simplex method used by Nguyen (1974) on ep is examined, and it is shown how this can be easily modified to deal with capacitated networks. (TRRL)

TRAFFIC EQUILIBRIUM METHODS: STATE-OF-THE-ART PAPERS

The International Symposium on Traffic Equilibrium Methods (Montreal University,

Quebec). Centre de Recherche sur les Transports (Montreal University). Sponsored by the Urban Mass Transportation Administration, Washington, D.C. Report No. UMTA-FN-06-0001-75-1 (January 1975): 260.

Twelve papers, selected from those presented at the International Symposium on Traffic Equilibrium, summarize the current state of model research, computation and practice in this area. The papers are on topics as follows: Equilibrium versus Optimum in Public Transportation Systems; Integrated Equilibrium Flow Models for Transportation Planning; Equilibrium Methods for the Study of a Congested Urban Area; Area Traffic Control and Network Equilibrium; Delay Functions in Assignment Problems; Analysis and Comparison of Behavioral Assumptions in Traffic Assignment; Supply Functions for Public Transport-Initial Concepts and Models; Equilibrium Methods for Traffic Assignment; Freeway Corridor Assignment Equilibria; Network Equilibrium Capabilities for the UMTA Transportation Planning System; The Choice of Assignment Techniques for Large Networks; Equilibrium Models in Use-Practical Problems and Proposals for Transport Planning.

TRAFFIC EQUILIBRIA ANALYZED VIA GEOMETRIC PROGRAMMING

Hall, Michael A. and Elmor L. Peterson (Northwestern University; Evanston, IL). Dept. of Industrial Engineering and Management Sciences. Sponsored by the Air Force Office of Scientific Research, Arlington, VA. Report No. AFOSR-TR-75-0549 (February 1975): 58.

The 'traffic-assignment problem' consists of predicting 'Wardrop-equilibrium flows' on a roadway network when origin-to-destination 'input flows' are specified. The 'demand-equilibrium problem' consists of predicting those input flows that place the network in a state of 'economic equilibrium' when the input flows are related via given travel-demand (feedback) curves to the resulting Wardrop-equilibrium origin-to-destination 'travel costs.' The traffic-assignment problem is treated as a special case of the demand-equilibrium (the case in which the travel-demand curves are graphs of constant functions); and the demand-equilibrium problem is formulated and studied in the context of (generalized) 'geometric programming.' In particular, existence, uniqueness and characterization theorems are obtained via the duality theory of geometric programming by introducing appropriate extremality conditions and their corresponding dual variational principles (sometimes called complementary variational principles). These dual variational principles and extremality conditions also lead to new computational algorithms that show promise in the analysis of relatively large networks (such as those in relatively large urban or metropolitan areas).

Related Articles

THE FRANK-WOLFE ALGORITHM FOR EQUILIBRIUM TRAFFIC ASSIGNMENT VIEWED AS A VARIATIONAL INEQUALITY

Van Vliet, D. (University of Leeds). Transportation Research. Part B: Methodological 21B, No. 1 (February 1987): 87-89.

AN APPLICATION OF OPTIMAL CONTROL THEORY TO DYNAMIC USER EQUILIBRIUM TRAFFIC ASSIGNMENT

Wie, B. (National Research Council; Washington, D.C.). Transportation Research. (1988): 30.

ROAD TRAFFIC ASSIGNMENT: A REVIEW II. EQUILIBRIUM METHODS

Matsoukis, E.C., and P.C. Michalopoulos (University of Minnesota). Transportation Planning and Technology 11, No. 2 (August 1986): 117-135.

THE USE OF QUANTAL LOADING IN EQUILIBRIUM TRAFFIC ASSIGNMENT

Arezki, Y., and D. Van Vliet (University of Leeds). Transportation Research. Part B: Methodological 19B, No. 6 (December 1985): 521-52.

AN EFFICIENT IMPLEMENTATION OF THE "PARTAN" VARIANT OF THE LINEAR APPROXIMATION METHOD FOR THE NETWORK EQUILIBRIUM PROBLEM

Florian, M.A., Guelat, J., and H. Spiess (Universite De Montreal). Centre De Recherche. N395 (1985).

EQUILIBRIUM FLOWS IN A NETWORK WITH CONGESTED LINKS

Okutani, I. The 9th International Symposium on Transportation and Traffic Theory (Delft, the Netherlands). Transportation and Traffic Theory, (July 1984).

DECOMPOSITION TECHNIQUES FOR THE TRAFFIC ASSIGNMENT PROBLEM

Lawphongpanich, S. (University of Florida). Ph.D. Thesis. (1983).

CONVEX PROGRAMMING FORMULATIONS OF THE ASYMMETRIC TRAFFIC ASSIGNMENT PROBLEM

Hearn, D.W., S. Lawphongpanich, and S. Nguyen (University of Florida). Transportation Research. Part B: Methodological 18B, No. 4/5 (October 1984).

COMBINATIONS OF VARIOUS MODELS

This group of references includes methods that are derived from two types of combinations:

- (1) combinations of conventional transportation planning models, i.e., trip generation, mode choice, trip distribution and traffic assignment, and
- (2) combinations of traffic assignment models, i.e., minimum path assignment, capacity-restraint assignment, multipath assignment and equilibrium assignment.

ON THE RELATIONSHIP BETWEEN THE DISCRETE AND CONTINUOUS MODELS FOR COMBINED DISTRIBUTION AND ASSIGNMENT

Erlander, S. Transportation Research. Part B: Methodological 22B, No. 5 (October 1988): 371-382.

The relationship between the discrete and continuous models for the combined distribution and assignment problem is presented. In particular, it is shown that, under certain conditions, the classic continuous flow formulation of the combined distribution and assignment problems can be viewed as the continuous approximation of the discrete model formulated in terms of the Efficiency Principle.

THE ESTIMATION OF A COMBINED DISTRIBUTION ASSIGNMENT MODEL FROM TRAFFIC COUNTS

Scherders, L. (Verkeersacademie Tilburg Stappegoorweg; Tilburg, the Netherlands). (1981): 99, Tabs. 27.

The aim of a traffic and transport model is to determine the quantity of traffic on road sections and the distribution of the traffic on a road network. The main purpose of this report is the estimation of a combined distribution/assignment model. The aim of this type of model is to calculate the origin-destination matrix from a small amount of intensities on road sections which is the basis of a measured traffic load. This calculated origin-destination matrix can be assigned to the road network. By this the traffic load on each road section is known. The distribution function is determined in such a way that it links on a preset trip pattern. The problem is, however, that the distribution matrix is not determined unambiguously by a complete trip pattern. Two model techniques are available: (1) the gravity model and (2) the entropy maximum likelihood model. In this report a combination of these two models is presented which retains the advantages of both of them. (TRRL)

COMPUTATIONAL EXPERIENCE WITH AN APPLICATION OF A SIMULTANEOUS TRANSPORTATION EQUILIBRIUM MODEL TO URBAN TRAVEL IN AUSTIN, TEXAS

Monographic, not a series, International Symposium.

Safwat, K.N.A., and C.M. Walton. Transportation Research. Part B: Methodological 22B, No. 6 (December 1988): 457-467.

Safwat and Magnanti (1988) have developed a combined trip generation, trip distribution, modal split and traffic assignment model that can predict demand and performance levels on large-scale transportation networks simultaneously, i.e., a simultaneous transportation equilibrium model (STEM). The major objective of this paper is to assess the computational efficiency of the STEM approach when applied to an urban large-scale network, namely the urban transportation system of Austin, Texas. The Austin network consisted of 520 zones, 19,214 origin-destination (O-D) pairs, 7,096 links and 2,137 nodes.

The Central Processing Unit (CPU) time on an IBM 4381 mainframe computer was 430 seconds for a typical iteration and about 4,734 seconds, less than 79 minutes, to arrive at a reasonably accurate solution in no more than 10 iterations. Computational time at any given iteration is comparable to that of the standard fixed demand traffic assignment procedure. These results encourage further applications of the STEM model to large urban areas.

COMPUTATIONAL EXPERIENCE WITH A CONVERGENT ALGORITHM FOR THE SIMULTANEOUS PREDICTION OF TRANSPORTATION EQUILIBRIUM

Safwat, K.N.A. *Transportation Logistics*. Transportation Research Record No. 1120 (1987): 60-67, Fig. 3, Tab. 2.

A report is given of the computational experience with a globally convergent algorithm (Shortest Path to the most Needy Destination - SPND) that predicts trip generation, trip distribution, modal split, and traffic assignment simultaneously on a Simultaneous Transportation Equilibrium Model, developed by Safwat and Magnanti, when it is applied to analyze intercity passenger travel in Egypt. A good convergence criterion, known a priori to be zero at equilibrium, was found on the basis of the solution procedure itself. In order to achieve an accuracy of about 1 percent within the optimum value of the objective function, the CPU time on a VAX-11 VMS computer was 379 sec. for a network with 24 origins, 552 origin-destination pairs, 152 nodes, and 224 links. The SPND algorithm is expected to perform better in applications involving the usual urban traffic congestion in contrast to the "fictitious severe congestion" caused by the existence of fleet capacity constraints on the Egyptian intercity system. A companion paper by Safwat in this Record (Transportation Research Record No. 1120, pp 52-59) addresses the behavioral aspects of the application.

A COMBINED TRIP GENERATION, TRIP DISTRIBUTION, MODAL SPLIT, AND TRIP ASSIGNMENT MODEL

Safwat, K.N.A., and T.L. Magnanti. Transportation Science 22, No. 1 (February 1988): 14-30.

Modelling of transportation systems must invariably balance behavioral richness and computational tractability. In this paper, a transportation equilibrium model and an algorithm for the simultaneous prediction of trip generation, trip distribution, modal split and trip assignment on large-scale networks is developed. The model achieves a practical compromise between behavioral and computational aspects of modelling the problem. It is formulated as an equivalent convex optimization problem, yet it is behaviorally richer than other models that can be cast as equivalent convex programs. Although the model is not as behaviorally rich as the most general equilibrium models, it has computational advantages. The applications reported in this paper of the model to real systems, i.e., the intercity transport network of Egypt and the urban transportation network of Austin, Texas,

illustrate the computational attractiveness of the approach. These results indicate that equivalent optimization formulations are not as restrictive as previously thought; hence, the equivalent convex programming approach for modelling and predicting equilibrium on transportation systems remains a viable and fruitful avenue for future consideration.

ON BINARY MODE CHOICE/ASSIGNMENT MODELS

Florian, M., and H. Spiess (Montreal University, Canada). Transportation Science 17, No. 1 (February 1983): 32-47.

The paper considers the two mode equilibrium road and transit assignment models which incorporate a zonal aggregate mode choice model. The paper reformulates this special structure network equilibrium model as a variational inequality and demonstrates that the origin to destination demands and travel costs, link flows and link travel costs are unique when appropriate sufficient conditions are satisfied. The existence of equivalent optimization formulations of special versions of this problem are demonstrated.

BEHAVIOR-ORIENTATED MODELS AS THE BASIS OF URBAN TRANSPORT PLANNING

Leutzbach, W. (Karlsruhe University, West Germany). Deutschen Verkehrswissenschaft Gesellschaft E.V. Schriftenreihe Deutschen Verkehrswiss Ges, R B Sem N45 (January 1980): 53-60. Tab. 4.

In recent times an attempt has been made to use behavior-orientated models as the basis of urban transport planning in addition to the already proven gravity models. Behavior-orientated models combine traffic generation, traffic distribution and choice of transport mode as stages in the four stage algorithm into a single simulation sequence. Only route selection must be carried out separately for pragmatic reasons. The advantages of the behavior-orientated models lie in the possibility of being able to quantify the effects of certain behavioral changes or changes in the structure of availability on traffic behavior and traffic demand. (TRRL)

INTERACTION BETWEEN LAND-USE/DISTRIBUTION AND ASSIGNMENT

Luk, J.Y.K., and R.J. Nairn (Australian Road Research Board; Nairn (RJ) and Partners Pty Limited). Engineering Conference (Adelaide). National Conference Publication No. 80/2 (1980): 195-201, Fig. 10.

This paper reports on a study to investigate the feasibility of determining a synthetic equilibrium between the demand for travel and level of service provided by a network. The method adopted is an iterative algorithm that involves the feedback of a travel cost matrix, obtained at the end of a traffic assignment phase, to a land-use/distribution phase. TRANSTEP, a land-use/distribution model, is used for modelling travel demand, and

ARRBTRAFIC, an equilibrium assignment model, is used for estimating travel delays between each pair of origin-destination zones. The algorithm is tested using three different initial conditions. The results indicate the existence of an equilibrium region after four to five iterations. Further iterations lead to oscillations with an amplitude of five to ten percent around the equilibrium. These interim results demonstrate the effect of congestion costs on demand models such as TRANSTEP and identify areas suitable for the refinement of both TRANSTEP and ARRBTRAFIC. (Author/TRRL)

UTPS TECHNICAL BRIEF: LINKING THE UTPP AND UTPS

Clarke, M.B. (Comsis Corporation; Wheaton, MD) Urban Mass Transportation Administration Office of Technical Assistance, Washington D.C. Report No. UMTA-URT-41-83-13 (September 1983): 50.

The purpose of this paper is to aid the transportation planner in extracting data from the UTPP (Urban Transportation Planning Package) for use in computer-assisted urban travel modelling. The paper is divided into several sections. The first is a general description of what the user will be confronted with when he or she receives the UTPP package. This section details procedures on how to access and use the UTPP print program. It also compares the UTPP with the past data handling methods contained in the UTPS program UCEN70. The next section discusses how to reformat the UTPP data for UTPS (Urban Transportation Planning System) use and for use in microcomputer software packages. The third section discusses the creation of UTPS Z-file (Zonal Data) and J-file (Trip Interchange) data structures. Building these structures is complicated by the volume of UTPS documentation, so this section provides a short and straightforward approach to mastering the process. The fourth section comprises a major portion of the paper. In it, examples are presented showing how the UTPP data can be used in trip generation, trip distribution, modal choice, and traffic assignment. All examples show the use of UTPS in the travel demand modelling process. Following this section, a discussion of how UTPP data can be "downloaded" from a mainframe to a microcomputer is presented. In addition, this section gives examples of microcomputer applications.

UTPS TECHNICAL BRIEFS: TAMPA SYMPOSIUM HIGHLIGHTS

Price, Williams and Associates, Incorporated (Silver Spring, MD). Urban Mass Transportation Administration, Washington D.C. Report No. UMTA-MD-06-0100-84-1 (November 1984): 58.

The State of Florida has been more active than most in providing technical assistance to local governments. Florida also has a wealth of experienced UTPS users who have further improved the system with software of their own. Thus, Florida was chosen as the site of the State Government UTPS Users' Forum held from May 29 to June 1, 1984, at the Caribbean Gulf Resort in Clearwater Beach. This document is a record of those proceedings and is being distributed so that others may profit from the information presented. Forum

discussions were not limited to just UTPS-related matters but also covered such topics as microcomputer usage and organizational responsibilities. Such diversity of information is reflected accordingly in this document. The first section gives the reader an overview of Florida's own planning philosophy. The second section is more technical and deals with both microcomputer and UTPS programs. The third and final section is composed of discussions of administrative and organizational concerns. Other pertinent information and a UTPS order form can be found at the back of the document.

METHODS FOR COMBINING MODAL SPLIT AND EQUILIBRIUM ASSIGNMENT MODELS

Abdulaal, M., and L.J. LeBlanc (Southern Methodist University). Transportation Science 13, No. 4 (November 1979): 292-314.

Two ways for combining modal split and traffic assignment techniques into simultaneous mode and route choice models are discussed. In one model, users are assumed to choose their routes according to J. G. Wardrop's equilibrium principle and their modes according to mode choice functions of the difference in modal disutility. In the other model, a route and mode choice equilibrium in which no user can change his route or mode to lessen his general disutility is assumed. The advantages and disadvantages of each of these combined models are discussed.

A STUDY OF A DECOMPOSITION BY ORIGIN METHOD FOR THE COMBINED DISTRIBUTION AND ASSIGNMENT PROBLEM

Joernsten, K.O. (Linköping University, Sweden). Matematiska Institutionen. S-58183 (1979): 16, Fig. 7, Tab. 4.

This paper investigates the suggested alternative Dantzig-Wolfe decomposition method for the system optimized combined distribution and assignment problem given by Tomlin (1971). The problem can be formulated in many ways. Tomlin (1971) uses an objective function which is a weighted function between total travel cost and interactivity (entropy); an alternative formulation with an interactivity objective and total cost constraint can also be used. In this paper the second formulation with constant link costs and explicit capacity constraints is used. The second part of the paper gives computational comparisons for the alternative decomposition methods, decomposition by origin (suggested in Tomlin, 1971) and Bender's decomposition, Joernsten (1979). (TRRL)

ON THE CALIBRATION OF THE COMBINED DISTRIBUTION-ASSIGNMENT MODEL

Erlander, S., S. Nguyen, and N.F. Stewart (Linköping Institute Of Technology, Sweden; Montreal University, Canada). Transportation Research. Part B: Methodological 13B, No. 3 (September 1979): 259-267, Fig. 1.

In this article a model is considered which combines gravity model trip distribution and equilibrium assignment. This model contains a free parameter which must be fixed using supplementary information, that is, the model must be calibrated. It is natural to ask which supplementary information is sufficient to determine the free parameter. Here it is shown that under very weak hypotheses each of the quantities, the entropy h or the total assignment cost f , is alone sufficient to determine the free parameter. In contrast, it is shown by means of a counterexample that even under very strong hypotheses, the total cost c is not sufficient. (TRRL)

EQUILIBRIUM SOLUTIONS TO COMBINED URBAN RESIDENTIAL LOCATION, MODAL CHOICE, AND TRIP ASSIGNMENT MODELS

Boyce, D.E. (Regional Science Research Institute; Philadelphia, PA). (May 1977): 32.

Urban land use and transportation models have historically incorporated the effects of highway capacity and congestion in a conceptual sense; however, methods for solving these models that satisfy Wardrop's conditions of user-equilibrium have been developed only recently. The application of these convergent equilibrium methods to Lowry-type residential location models provides a means for determining the spatial distribution of residential activities which is in equilibrium with multi-model transportation costs.

A COMBINED TRIP DISTRIBUTION MODAL SPLIT AND TRIP ASSIGNMENT MODEL

Florian, M., and S. Nguyen. Transportation Research 12, No. 4 (August 1978): 241-246.

A trip distribution, modal split and trip assignment model is developed. As is well known, the entropy type distribution model is equivalent to postulating a travel demand function where trips are proportional to a negative exponential function of the travel cost. It is shown that when two distribution models of this type are linked with route choice models based on Wardrop's "user optimized" principle, the mode choice is given by a logit model. A solution algorithm is developed that computes the resulting trip interchanges and route flows. (TRRL)

TRAFFIC ASSIGNMENT TECHNIQUES AT THE OPERATIONAL LEVEL

SPEEDING UP QUICKEST-ROUTE ASSIGNMENTS IN CONTRAM WITH AN HEURISTIC ALGORITHM

Taylor, N.B. Traffic Engineering and Control 30, No. 2 (February 1989): 70-2.

The contram traffic assignment program uses a variation of Dijkstra's quickest-route algorithm for finding routes between particular origins and destinations. The run-time of contram is dominated by the large number of vehicle assignments which is to perform, so the efficiency of the route algorithm is critical. This paper describes a practical heuristic method of improving the efficiency of the quickest-route assignment algorithm by using simple estimates of the travel times between all network nodes and each destination. Examples are given which show that the amount of computation can be reduced by as much as two-thirds. (Author)

DYNAMIC INTEGRATED FREEWAY/TRAFFIC SIGNAL NETWORKS: A ROUTING-BASED MODELLING APPROACH

Van Aerde, M. and S. Yagar. Transportation Research. Part A: General 22A, No. 6 (November 1988): 445-453, Fig. 2, Tab. 1.

While the potential benefits of integrating freeway and traffic signal control have long been recognized, fundamental incompatibilities between their respective modelling approaches have prevented the development of a suitable simulation mode. Such a model is not only required to evaluate the potential of externally generated control strategies but should ultimately also form the basis for a comprehensive optimization model. This paper presents a new modelling approach, called INTEGRATION, which has been designed specifically to evaluate such integrated networks and their controls. The approach considers the behavior of traffic flows in terms of individual vehicles that have self-assignment capabilities. This capability serves as a traffic assignment function within an integrated network of traffic signals and freeways and circumvents the need to use either an explicit time slice or iterations during the traffic assignment. Consequently, one can consider continuously variable traffic demands and controls, both freeway and signalized networks, as well as any links that join them. While the proposed approach requires further field testing and validation, it does represent a critical step towards the development of an ultimate tool for evaluating and optimizing integrated traffic networks and their controls.

INTERSECTION ANALYSIS USERS' GUIDE

Hamburg (John) and Associates, Inc. and Price, Williams and Associates, Inc. (Spring, MD; Urban Mass Transportation Administration, Washington, D.C.) Report No. UMTA-URT-41-84 (March 1984): 157 Fig., Tab. 5.

Traffic assignment with intersection analysis is a highway transportation planning tool which permits treatment of delays due to congestion at intersections in far greater detail than is possible with conventional traffic assignment techniques. Consequently, it greatly increases the range of problems which can be studied within the context of traffic assignment. Intersection Analysis operates as a submodel within the UTPS highway traffic assignment program UROAD. Intersections for detailed assignment treatment are identified by the program user; additional data describing the geometry and functional characteristics of these intersections are input to the program. The Intersection Analysis Users' Guide presents a rationale for traffic assignment at the micro level, a detailed theoretical development of the delay algorithms, a complete description of required coding procedures, and a set of illustrative examples.

PROCEDURE FOR PREDICTING QUEUES AND DELAYS ON EXPRESSWAYS IN URBAN CORE AREAS

Lisco, T.E. Transportation Research Board, Transportation Research Record N944 (1983): 148-154, Fig. 9.

TRAFFIC ASSIGNMENT AT THE OPERATIONS LEVEL: THE VERIFICATION AND APPLICATION OF CONTRAM IN EDMONTON

Stephenson, B. and S. Teply (GCG Engineering Partnership, Alberta University, Canada). The 8th Annual Conference on New Directions in Transportation Engineering (Edmonton, Alberta, Canada). Institute of Transportation Engineers. (1983): 20, Fig. 13.

In 1977, a study of the traffic impact of the future development of the eastern portion of Edmonton's downtown was initiated. Trip generation was derived from the area's development potential, a traditional traffic assignment model used to obtain the volumes and the TRANSYT signal optimization program employed for the evaluation of the network performance. The comparison of traffic assignment and TRANSYT results revealed that significant problems existed. Assigned volumes produced unacceptable delays at certain intersections while parallel routes were underutilized. In 1979, the University of Alberta obtained a pre-release version of CONTRAM to test the program performance and to provide feedback. At that time, the University, with the support of the city of Edmonton, was conducting research analyzing the adaptability of transportation systems with a special focus on the impact of transportation systems management measures. In 1981-1982, two major studies utilized the results of the previous research and further tested CONTRAM. Both studies directly related to the Edmonton downtown. Both the research and the applications have shown that CONTRAM is a valuable analytical and design tool. This paper discusses the results of the program verification and briefly describes the applications in Edmonton. (TRRL)

SATURN - A MODERN ASSIGNMENT MODEL

Van Vliet, D. (Leeds University, England). Traffic Engineering and Control 23, No. 12 (December 1982): 578-581, Fig. 3.

SATURN (Simulation and Assignment of Traffic to Urban Road Networks) is a computer model for the analysis and evaluation of traffic management schemes over relatively localised networks. It serves, in effect, as a highly sophisticated traffic assignment model. It was first described in Traffic Engineering & Control in April 1980 and more recently, in brief, in July/August 1982. This article updates these previous ones by describing recent improvements to the model: two levels of network description to enable wide-ranging impacts to be studied, fuel consumption estimates, explicit bus routes, a correct treatment of over-capacity junctions, a quasi-dynamical assignment facility, graphical outputs, interactive analysis via a VDU, plus a number of miscellaneous improvements. (Author/TRRL)

USER GUIDE TO CONTRAM VERSION 4

Leonard, D.R. and P. Gower (Transport and Road Research Laboratory, Berkshire, England). Report No. SR 735 (1982): 68, Fig. 4.

Contram is a traffic assignment model for use in the design of traffic management schemes. The model predicts vehicle routes, flows and queues in a network of streets and junctions; junctions may be controlled by traffic signals or "give-way" rules. It is assumed that the numbers of trips between each origin and destination are known and that they may vary with time so that the growth and decay of congestion in peak periods can be studied. Allowance is made for the physical size of queues which may block back and restrict the throughput capacity at upstream junctions. Up to three classes of vehicles (e.g. cars, buses and lorries) can be represented, and selected vehicles (usually buses) can be sent along fixed routes. Contram provides comprehensive information on traffic conditions including link delay times, turning movements, and fuel consumption to help the traffic engineer to understand and assess the merits of alternative traffic management schemes. This user guide gives a brief description of the model, details of data input and output, a test example to illustrate the use of the model and an outline of the structure and operation of the computer program. (Author/TRRL)

TRAFLO: A NEW TOOL TO EVALUATE TRANSPORTATION SYSTEM MANAGEMENT STRATEGIES

Lieberman, E.B. and B.J. Andrews (KLD Associates Incorporated). Transportation Research Board. Transportation Research Record N772 (1980): 9-15, Fig. 3, Tab. 1.

The TRAFLO model, which combines the attributes of traffic simulation with traffic assignment, is described. TRAFLO was developed as a tool for use in transportation planning and traffic engineering to test transportation management strategies. It is a

software system, programmed in FORTRAN, that consists of five component models that interface with one another to form an integrated system. Four of the models simulate traffic operations, and the fifth is an equilibrium traffic assignment model. The operating characteristics of the component simulation models are described. These models are capable of simulating traffic on one or more of the following networks: freeways, corridors that include the freeway/ramp/service-road complex, urban and suburban arterials, and grid networks representing the central business districts of urban centers. Also described is the traffic assignment component which can be used in conjunction with the simulation components to determine the response of a traffic system to a transportation management strategy. (Author)

A COMPARATIVE STUDY OF THREE URBAN NETWORK MODELS: SATURN, TRANSYT/8 AND NETSIM

Luk, J.Y.K. and R.W. Stewart (Australian Road Research Board; Waterloo University, Canada). The 9th Australian Transport Research Forum (Adelaide, Australia). South Australia Department of Transportation (1984): 51-66. Fig. 2, Tab. 4.

The experience of using three urban network models for predicting the performance of signal co-ordination in an arterial road is reported. The three models, or packages, selected are SATURN, TRANSYT/8 and NETSIM. They represent different levels of modelling details. SATURN has traffic assignment capability, but this aspect is not investigated in this paper. The three models were tested using a real-world network in Parramatta, New South Wales. The task of preparing three sets of consistent input data was found to be non-trivial. Several problems were encountered and could be attributed to incompatible structures in these models. For example, the simulation module of SATURN does not directly accept link traffic flows as input data. Saturn was found to underestimate total delay by twenty-nine percent and fuel consumption by seven percent when compared with TRANSYT/8. There was, however, no difference between the two models in the prediction of the number of stops. NETSIM was found to have a bias in the lane distribution of traffic flows. This problem frequently created spill-back, and the model was difficult to use in near-saturated conditions. TRANSYT/8 was found to be the simplest of the three models in preparing the input data and the most consistent in performance prediction. (Author/TRRL)

SIMULATION STUDIES FOR AN URBAN TRAFFIC CORRIDOR.

McCabe, L., R. Ricci, and P. Concannon. Department of Transportation, Transportation Systems Center. Sponsored by the Federal Highway Administration. (1975): 47.

The salient features of the Simulation of Corridor Traffic (SCOT) model and a successful calibration and validation are described. SCOT is a computer model applicable to an urban traffic corridor and capable of simulating vehicular traffic on freeways, including on and off ramps and urban streets. Vehicles are treated microscopically on the arterial street system and macroscopically as platoons on the freeway. Output statistics for each simulated link

include numbers of vehicles discharged, total times of travel, average occupancy and average speeds for specified time intervals. Calibration and validation data collected using photographic techniques on a 1.2 mile test network of the Dallas North Central Expressway are described. Tests showed no significant differences between field and simulation results at the 1% level of significance for the basic parameters of traffic (mean speed, flow and saturation). This and other applied tests indicate that the SCOT model adequately replicates freeway traffic performance. A demonstration of the origin-destination (O-D) traffic assignment capability of the model is described (using license plate data recorded at the frontage road and at each ramp on the test network). It is indicated that the minimum time-path criteria used have not been conclusively shown to be the correct criteria for origin-destination traffic assignments. An analysis of freeway ramp control versus no ramp control for the Dallas test network shows a statistical reduction in speed with no ramp control for those freeway links in the vicinity of the on-ramp.

VALIDATION WORK ON CONTRAM - A MODEL FOR USE IN THE DESIGN OF TRAFFIC MANAGEMENT SCHEMES

Leonard, D.R. and J.B. Tough (Transport and Road Research Laboratory). PTRC Annual Summer Meeting. PTRC Education and Research Services Ltd. (1979): 135-153, Fig. 6, Tab. 4.

The paper describes validation work on the traffic assignment model CONTRAM which is being developed for use in the design of traffic management schemes. CONTRAM can predict the flows and queues of vehicles in a network during peak periods and can take into account the effects of oversaturated conditions which may occur during these periods. The model assumes that drivers have a good knowledge of traffic conditions and choose minimum journey time routes through the network. Comprehensive data on traffic movements were collected in the town of Reading for morning and evening peak periods using a number plate survey. The data were analyzed to provide time-varying origin-destination demands, flows, turning movements, journey times and routes; and comparisons were carried out with the traffic patterns predicted by the model. Day to day variations in traffic patterns were also examined. The results suggest that CONTRAM is a realistic model. It is shown that observed patterns of flows, turning movements and journey times are reasonably stable from day to day and that the model predictions generally lie within the day to day variations. (Author/TRRL)

SATURN: A MODEL FOR THE EVALUATION OF TRAFFIC MANAGEMENT SCHEMES
Bolland, J.D. and M.D. Hall (Leeds University, England). Institute for Transportation Studies. (January 1979): 58, Fig. 21.

The model SATURN (Simulation and Assignment of Traffic in Urban Road Networks) may be considered as a highly enhanced assignment model, investigating as it does the

effects of traffic management on flows in the network. The enhancement arises from the use of a simulation model which, as well as providing information for the assignment on the variation of delay with flow for each turn, can be used to assess the more general impacts of traffic such as noise. Because of the critical nature of the delays at intersections, these are modelled in considerable detail. The purpose of this paper is to describe in detail the structure, the assumptions made, the data requirements and the possible output from the model. After describing the assignment-simulation loop which is the basis of SATURN, it then describes the two stages in detail. The running of the complete model is summarized and details given of the practical experience of its use to date. Finally, some possible future developments are mentioned. (Author/TRRL)

DYNAMIC STOCHASTIC CONTROL OF FREEWAY CORRIDOR SYSTEMS SUMMARY AND PROJECTIVE OVERVIEW

Athans, M., S.B. Gershwin, and P.K. Houpt (Massachusetts Institute of Technology, Cambridge, MA). Electronics Systems Laboratory. Sponsored by the Department of Transportation, the Research and Special Programs Administration, and the Transportation Systems Center. (December 1978): 74.

Systematic methodological approaches to overall traffic management from both short-term (real-time) and long-term (planning) perspectives have been developed. The approach embodies formulation and solution of interrelated mathematical problems from operations research and optimal control theory which includes the following: (1) selection of traffic models, both static and dynamic, (2) optimal (steady-state) traffic assignment, (3) dynamic ramp-metering control algorithm design, and (4) traffic surveillance data-processing algorithm design. Theoretical results and designs are evaluated using a variety of computer simulation tools. Practical implementation of these procedures for traffic management are addressed in all phases of the research through decomposition and decentralization techniques.

A TRAVEL TIME MODEL FOR URBAN ROAD NETWORKS: A DETAILED DESCRIPTION OF PRINCIPLES AND PROCEDURES

Bovy, P.H.L. and G.R.M. Jansen (Delft University of Technology, the Netherlands). Institute for Town Planning Research. (1977): 87.

It is proposed to use a travel time model in order to estimate travel times in urban road networks. A model is presented to derive time flow functions for assignment purposes. Travel time is related to link and intersection characteristics, traffic flow characteristics, behavioral parameters, etc. Basic to the approach is the derivation of separate travel time functions for links and intersections, subsequently to be combined into one function for the complete road section. Intersection delays are assumed to play the major role in total travel time loss. Intersection delays are estimated using stochastic waiting models. The use of the

model in transport planning practice is set out and is illustrated with numerical examples. (TRRL)

DYNAMISCHE UMLEGUNG IN VERKEHRSNETZEN. (DYNAMIC TRAFFIC ASSIGNMENT IN TRAFFIC NETWORKS)

Leutzbach, W., M. Haas, V. Papavasiliou, and T. Schwerdtfeger (Karlsruhe University, Federal Republic of Germany). Inst. fuer Verkehrswesen 469 (1986): 21.

In a traffic assignment scheme, a trip matrix given for a specific means of transport is assigned to a traffic network. The conventional methods are static, i.e. they are not capable of dealing with dynamic changes in the trip matrix. Methods which are capable of taking such dynamic instationarities into account are called "dynamic assignment methods." For this study, a dynamic assignment model was developed on the basis of a model for the simulation of the traffic flow on expressway networks. The model simulates the traffic flow by the movement of individual vehicles through the network at discreet intervals. The simulation model was extended by one module which, at intervals to be determined, calculates the routes for which the shortest time is needed and which are then used by the vehicles during the next interval. The developed model was demonstrated by means of a real network example. In order to simplify the procedure, however, the trip matrix used was established on the basis of pragmatic assumptions, because this is sufficient for the demonstration of a model. Four simulation cycles were carried out. In this connection, the influence of the intervals for the determination of the shortest routes on the result of the assignment was examined. It was found that smaller intervals between the recalculation of the shortest routes result in a considerably more balanced traffic situation within the network than with longer intervals. The objective of the research activities was to develop an instrument allowing traffic flows on an expressway network to be assigned dynamically, i.e., as a function of time, to this network. The question as to which assignment intervals will bring about realistic results for specific network structures and traffic load situations would have to be examined in follow-up studies. (Author)

MUTUALLY CONSISTENT TRAFFIC ASSIGNMENT AND SIGNAL TIMINGS FOR A SIGNAL-CONTROLLED ROAD NETWORK

Charlesworth, J.A. (Newcastle Upon Tyne University, England). Transport Operations Research Group. Report No. RR-24 (January 1977): 44.

Use has been made of a general procedure for estimating the relationships between travel-time and traffic flow for all the links of a signal-controlled road network for any given signal timings in a form suitable for use in assignment calculations. The techniques of traffic control and traffic assignment have then been combined to enable the calculation of mutually consistent signal timings and traffic assignment by means of an iterative procedure. There will usually be, in principle, many solutions to such calculations for a given network and given origin-destination movements, and a case is described in which two

quite distinct solutions have been found for a small realistic network, using as a starting point in each instance some preconceived initial assignment of traffic.

CONTRAM: A TRAFFIC ASSIGNMENT MODEL FOR PREDICTING FLOWS AND QUEUES DURING PEAK PERIODS

Leonard, D.R., J.B. Tough, and P.C. Baguley (Transport and Research Laboratory, Crowthorne, England). Report No. TRRL-LR-841 (1978): 30.

The report describes a computer-based traffic assignment and queuing model for use in the design of urban traffic management schemes. The model is called CONTRAM - CONTinuous TRaffic Assignment Model. CONTRAM requires data on time-varying flow demands (for example as occur in a 'peak period'), and predicts the flows, delays and queues throughout a network of roads and junctions. The movement of traffic is modelled by grouping vehicles together to form 'packets.' Each packet is assigned to its minimum journey time route through the network, taking into account delays and queues at junctions. The report is divided into three parts. The first part outlines the assumptions and structure of the model. The development of the model, sensitivity tests and validation work are described in the second part, and examples to illustrate applications of the model are described in the third part. (Author)

DYNAMIC STOCHASTIC CONTROL OF FREEWAY CORRIDOR SYSTEMS: SUMMARY AND PROJECT OVERVIEW

Athans, M., S.B. Gershwin, and P.K. Houpt (Massachusetts Institute of Technology, Cambridge, MA). Electronic Systems Laboratory, sponsored by the Transportation Systems Center, Cambridge, MA. Report No. DOT/TSC/RSPA-78-12 (December 1978): 73.

The report provides a summary and overview of the results obtained in the completion of the Contract 'Application of Modern Control Theory to Urban Freeway Corridor Control.' The focus of this activity has been to develop systematic methodological approaches to overall traffic management from both short-term (real-time) and long-term (planning) perspectives. Our approach embodies formulation and solution of interrelated mathematical problems from operations research and optimal control theory, including: (1) selection of traffic models, both static and dynamic; (2) optimal (steady-state) traffic assignment; (3) dynamic ramp-metering control algorithm design; and (4) traffic surveillance data-processing algorithm design. Theoretical results and designs are evaluated using a variety of computer simulation tools. Practical implementation of these procedures for traffic management is addressed in all phases of the research through decomposition and decentralization techniques.

A MODEL FOR DYNAMIC TRAFFIC ASSIGNMENT IN CONGESTED NETWORKS WITH SHOCK WAVES

Brastow, Jr., W.C. (Stanford University, CA). Department of Industrial Engineering. Report No. RR-13 UMTA-CA-11-0008-73-10 (December 1973): 125.

A mathematical formulation of a dynamic, deterministic traffic assignment algorithm particularly applicable to congested networks is specified. The model lies between the traditional stochastic simulation models and the static, multicommodity flow formulations of the traffic assignment problem in level of detail. The following properties make the model appear to be useful for the investigation of time varying flows in congested networks of moderately large size. Exogenous demands for travel between trip origins and destinations are treated as piecewise-constant functions of time.

Related Articles

TRAFFIC CONTROL AND TRAFFIC ASSIGNMENT IN A SIGNAL-CONTROLLED NETWORK WITH QUEUING

Smith, M.J. (University of York). Elsevier. Transportation and Traffic Theory (1987): 61-77.

TRANSPORTATION NETWORK ANALYSIS: INTERSECTION ANALYSIS, USERS' GUIDE: A GUIDE TO THE INTERSECTION ANALYSIS FEATURES OF THE UTPS PROGRAM

Office of Technical Assistance, Urban Mass Transit Administration, U.S. Department of Transportation, Washington, D.C. (March 1984): 156.

OTHER ARTICLES RELATED TO THE TRAFFIC ASSIGNMENT TECHNIQUE

ON THE VECTOR ASSIGNMENT P-MEDIAN PROBLEM

Hooker, J.N., and R.S. Garfinkel. Transportation Science 23, No. 2 (May 1989): 139-140.

The vector assignment p-median problem allows nodes of a network to be served by a nonclosest facility. This Technical Note presents a counterexample to a proposition of J.R. Weaver and R.L. Church that an all-node solution always exists. It is shown that the proposition is true, however, if every node is served by closer facilities at least as often as by more distant facilities.

A STOCHASTIC PROCESS APPROACH TO THE ANALYSIS OF TEMPORAL DYNAMICS IN TRANSPORTATION NETWORKS

Cascetta, E. Transportation Research. Part B: Methodological 23B, No. 1 (February 1989): 1-17, Fig. 5, Tab. 3.

Equilibrium analyses of transportation networks are by their nature "static" with equilibrium configuration defined as "fixed" or "autoreflexive" points, i.e., flow patterns reproducing themselves on the basis of the assumptions made on users' behavior once reached by the system. In this paper, it is argued that no transportation system remains in the same state over successive periods because of the action of several causes (e.g., temporal fluctuation of level and composition of demand, users' choices, and travel costs). This implies that the sequence of states occupied by the system over successive epochs or times of similar characteristics (e.g., AM peak hour of working days) is the realization of a stochastic process, the type of which depend on, among other things, the choice mechanism followed by travelers. Stationarity of the stochastic process within fixed potential demand and network structures is considered to be a desirable property because it allows a flow pattern distribution to be associated to each demand-network system independently of its starting configuration and elapsed time. Furthermore, this stationarity makes it possible to define expected path and link flows and compare them with those of stochastic user equilibrium (SUE). In this paper, rather general sufficient conditions for the process stationarity are given, essentially calling for a "stable" choice mechanism of potential users. In the following, a particular model of temporal dynamics (STODYN), based upon a number of simplifying assumptions on users' behavior common to most assignment models, is described. Exact and approximate relationships between (STODYN) steady-state expected flows and SUE average flows are also analyzed both in the case of unique and multiple equilibria. The possible use of STODYN as an assignment model giving unique average flows along with their variances and covariances is then discussed. The model takes into account stochastic fluctuations of demand and can be easily extended to other "dimensions" such as distribution and modal choice. Some results of an empirical analysis comparing STODYN average flows with SUE and observed flows on two urban car networks are also reported.

THREE DIMENSIONAL ASSIGNMENT METHOD IN THE TIME SPACE

Hamerslag, R., and P.C.H. Opstal (Technische Hogeschool Delft, the Netherlands). Report No. REPT-87-49 B87312008 (1987): 25.

Vehicle routing to avoid bottlenecks is discussed. An assignment algorithm which uses space as a third dimension is presented. It helps overcome problems with models which assign vehicles to the whole route.

TRAVEL TIME MODEL FOR URBAN ROAD NETWORKS: A DETAILED DESCRIPTION OF PRINCIPLES AND PROCEDURES

Bovy, P.H.L., and G.R.M. Jansen (Technische Hogeschool Delft, the Netherlands; Netherlands Institut voor Stedebouwkindig Onderzoek). Report No. MEMO-15 (April 1987): 99.

It is proposed to use a travel time model in order to estimate travel times in urban road networks. A model is presented to derive time flow functions for assignment purposes. Travel time is related to link and intersection characteristics, traffic flow characteristics, behavioral parameters, etc. Basic to the approach is the derivation of separate travel time functions for links and intersections, subsequently to be combined into one function for the complete road section. Intersection delays are assumed to play the major role in total travel time loss. Intersection delays are estimated using stochastic waiting models. The use of the model in transport planning practice is set out and is illustrated with numerical examples.

ON COMBINING MAXIMUM ENTROPY TRIP MATRIX ESTIMATION WITH USER OPTIMAL ASSIGNMENT

Fisk, C.S. Transportation Research, Part B: Methodological 22B, No. 1 (February 1988): 69-79.

This note shows how the maximum entropy trip matrix estimation procedure incorporated in ME2 as proposed by Willumsen (Van Zuylen and Willumsen, 1980) can be combined with a user optimal assignment model such as SATURN (Van Vliet, 1982) into a single mathematical problem. The resulting problem has the form of a bilevel programming problem similar to matrix estimation methods proposed by Nguyen (1981). The mathematical framework for the user optimal assignment problem is summarized, followed by a description of the maximum entropy trip matrix estimation procedure proposed by Willumsen. The two models are then combined into a single problem, and a discussion follows on solution procedures. (Author/TRRL)

THE DUAL OF THE TRAFFIC ASSIGNMENT PROBLEM WITH ELASTIC DEMANDS
Carey, M. (Carnegie-Mellon University). Ion Centre, Cowper Works. Bumpus, Haldane and Maxwell Ltd. 19B, No. 3 (June 1985): 227-237, Fig. 2.

This article is concerned with the dual of the traffic assignment problem, and of the combined generation, distribution and assignment problem. The duals and duality relations for the arc-chain and node-arc formulations of the problem are derived using only the Kuhn-Tucker conditions for convex programs. This has the advantage of being more familiar to most readers than the conjugate function presentation which has been used elsewhere. (Author/TRRL)

THE ROADWAY ASSIGNMENT COMPARISON PROGRAM, RDSTRC

Martin, I.W., and J.V. Douglas (United Information Services). Developing Countries Seminar (University of Warwick, England). PTRC Education and Research Services Ltd. (1983): 93-100, Fig. 3.

It is sometimes necessary for the transportation planner to assess the implication of a change in tree building assumptions within the modelling process (e.g., the effect of building trees on the basis of time rather than distance). How stable is a particular assignment under these circumstances? Other factors such as a change in the time needed to traverse a length of road may also be involved. The assessment of the influence of such changes is currently often the result of arduous manual techniques or somewhat limited computer applications. A problem applying to both of these approaches concerns the need to completely examine the model output in fine detail before it is possible to determine whether or not major changes have been induced. This paper presents a test case involving sample output from the run of program RDSTRC. RDSTRC will perform an assignment comparison on two roadway loaded networks and produce user controlled diagnostics aimed at assisting the user to readily identify not only the degree but also the location of changing journey patterns from one assignment to another. It has been designed to avoid the unnecessary use of computer time by means of the provision of user controlled progressive levels of detail. Thus an overall impression of assignment stability may be obtained prior to producing full comparison printouts which could prove redundant if there were negligible or even massive variation. The paper demonstrates how an alteration of the trees indicates initially a significant change in the assignment leading to further examination via the subsequent options. This increased detail includes tables and histograms summarizing the major features and reference is made to the "change" matrix. This matrix may be output from the program and contains trip information relating only to journeys which change route between assignments. The possibilities for further use of this matrix is demonstrated. (Author/TRRL)

DRIVING EFFORTS AND URBAN ROUTE CHOICE

Stern, E., J. Tzelgov, and A. Henik (Ben-Gurion University of the Negev, Israel; British

Columbia University). Logistics and Transportation Review 19, No. 1 (1983): 67-79, Fig. 2.

Time-related variables are not the only ones to be considered in traffic assignment procedures. Driving efforts measured with a psychological scale have been found to be of considerable influence in the individual's route choice process. Driving efforts are more effective in short-link transportation systems (short either in time or in route length) characterized by relatively high driving speed. Directions toward the explicit measurement and integration of driving efforts in route choice models are put forward. (Author)

EFFICIENT ALGORITHMS FOR SOLVING ELASTIC DEMAND TRAFFIC ASSIGNMENT PROBLEMS AND MODE SPLIT-ASSIGNMENT PROBLEMS

LeBlanc, L.J., and K. Farhangian (Vanderbilt University, Nashville, TN). Transportation Science 15, No. 4 (November 1981): 306-317.

Different algorithms for solving elastic demand traffic assignment problems and mode split-assignment problems are discussed. Different ways to implement the Frank-Wolfe technique to solve these problems is noted and the most efficient implementation is given. S. Evan's procedure is discussed and compared with the Frank-Wolfe technique.

UTILITY, ENTROPY AND A "PARADOX" OF TRAFFIC FLOW

Florian, M. (Montreal University, Canada). Transportation Research, Part A: General 15A, No. 4 (July 1981): 327-330.

The relation between various deviations of the logit-based route choice model and the associated benefit measures are explored in the light of "another" paradox of traffic flow highlighted recently by Sheffi and Daganzo. (Author/TRRL)

SUCCESSIVE LINEAR OPTIMIZATION APPROACH TO THE DYNAMIC TRAFFIC ASSIGNMENT PROBLEM

Ho, J.K. (Brookhaven National Laboratory). Operations Research Society of America. Transportation Science 14, No. 4 (November 1980): 295-305.

A dynamic model for the optimal control of traffic flow over a network is considered. The model, which treats congestion explicitly in the flow equations, gives rise to nonlinear, nonconvex mathematical programming problems. It has been shown for a piecewise linear version of this model that a global optimum is contained in the set of optimal solutions of a certain linear program. This paper presents a sufficient condition for optimality which implies that a global optimum can be obtained by successively optimizing at most $N + 1$ objective functions for the linear program, where N is the number of time periods in the planning horizon.

POSSIBLE USES OF CONTINUOUS MODELLING IN URBAN TRANSPORT PLANNING

Blumenfeld, D.E. and J.G. Wardrop (General Motors Research Laboratories; University College, London). World Conference on Transport Research (Imperial College, London, England). Organization for Economic Cooperation and Development. 3 (1980): 1929-39, Tab. 2.

This paper presents a transport planning technique which reduces the complexity of the real situation to a few major characteristics, suitable for strategic planning and assessment in urban areas. The approach used is to treat the urban area as a physical continuum and to express travel demand and spatial patterns of movement in terms of continuous functions of position. Such an approach offers an efficient way of estimating the amount of travel in different parts of a city while also providing a qualitative description of the spatial patterns of travel. Alternative planning strategies can be examined analytically as well as numerically thus providing a basic understanding of the overall effects on the transport characteristics in a city. The technique also allows variations on basic planning strategies to be investigated with relatively little additional cost or effort. This paper presents some of the principal developments of continuous modelling and its applications in strategic transport planning and discusses the relative advantages and disadvantages of continuous modelling compared with the conventional zonal transportation survey techniques of origin-destination tables and traffic assignment to a network.

ESTIMATION OF TRIP TABLES FROM OBSERVED LINK VOLUMES

Turnquist, M. and Y. Gur (Cornell University; Urban Systems, Incorporated) Transportation Research Record N730 (1979): 1-7, Fig. 8, Tab. 1.

Traffic assignment studies, which are instrumental in transportation plan evaluation, require an origin destination trip table as input. A method of estimating the trip table from observed link volumes within the framework of small-area assignment is described. The technique is based on solution of an optimization problem and is compatible with equilibrium traffic assignment assumptions. It thus differs significantly from previous methods based on statistical estimation. Because it avoids expensive home-interview surveys and time-consuming work with trip generation and distribution models, the method provides an attractive and cost-effective alternative analysis procedure for small- and medium-sized communities. It is likely to be particularly effective in the evaluation of short-term, low-capital improvements in the transportation network. (Author)

TRAFFIC ASSIGNMENT IN A TWO-DIMENSIONAL CONTINUOUS REPRESENTATION OF A TRAFFIC NETWORK WITH FLOW-DEPENDENT SPEEDS

Buckley, D.J. Transportation Research. Part B: Methodological 13B, No. 2 (June 1979): 167-179, Fig. 13.

The work deals with the assignment of traffic to a two-dimensional continuous representation of a traffic network. An important aspect of the treatment is that the reciprocal of the speed on each road in the network is at all times a linear function of the flow on that road. This speed-flow relationship is generalized to two-dimensional space using travel intensities and taking account of road densities, so that there is direct dependence of speeds upon flows at all points regardless of their location. There is also dependence of flows upon speeds at all points because Wardrop's first assignment principle is adopted. That is, for a given O-D pair, journey times on all routes actually used are identical and less than journey times on all other possible routes. This results in the identification for each O-D pair of an "assignment zone," an area within which all trips between that O-D pair are made and beyond which no such trips are made. For a single O-D pair the assignment zone is identified by θ , the maximum angular divergence of a path from the straight line between O and D. Paths are then assumed to be bilinear so that for a single O-D pair the assignment zone is a parallelogram. Journey time, speeds, lateral displacement and other related quantities are obtained as functions of the flow Q between O and D. The work is extended to three O-D pairs located at the extremities of an equilateral triangle and four O-D pairs located at the corners of a square. At low flows these two configurations are trivial extensions of the single O-D pair problem because assignment zones do not overlap. At higher flows, account is taken of this tendency to overlap so that, although they do not overlap, they do touch, becoming kite-shaped. Origins and destinations are assumed to be at the periphery of small circles of arbitrary radius. The work is inelegant to the extent that it involves a numerical integration, but it is possible that this might eventually be circumvented. (Author/TRRL)

OPTIMALITY CONDITIONS FOR A DYNAMIC TRAFFIC ASSIGNMENT MODEL

Merchant, D.K., and G.L. Nemhauser (Stanford Research Institute). Transportation Science 12, No. 3 (August 1978): 200-207.

A dynamic traffic assignment model formulated as a nonlinear and nonconvex mathematical program is described. Necessary optimality conditions require equalization of certain marginal costs for all the paths that are being used, and these optimality conditions are shown to be a generalization of the optimality conditions of a conventional static traffic assignment problem. The behavior of the dynamic model under static demand conditions is also examined, and it is shown that in this case the model presented is a generalized version of a standard static model.

MODEL AND AN ALGORITHM FOR THE DYNAMIC TRAFFIC ASSIGNMENT PROBLEMS

Merchant, D.K., and G.L. Nemhauser (Stanford Research Institute). Transportation Science 12, No. 3 (August 1978): 183-199.

A discrete time model is presented for dynamic traffic assignment with a single destination.

Congestion is treated explicitly in the flow equations. The model is a nonlinear and nonconvex mathematical programming problem. A piecewise linear version of the model, with additional assumptions on the objective function, can be solved for a global optimum using a one-pass simplex algorithm; branch-and-bound is not required. The piecewise linear program has a staircase structure and can be solved by decomposition techniques or compactification methods for sparse matrices.

STOCHASTIC DYNAMIC TRAFFIC ASSIGNMENT PROBLEM

Birge, J.R. and J.K. Ho (Michigan University, Ann Arbor, MI). Dept. of Industrial and Operations Engineering. Report No. TR-87-16 (1987): 22.

This paper presents a method for solving a stochastic version of the dynamic traffic assignment problem. It shows that a globally optimal solution may be obtained by a sequence of linear optimizations. A decomposition algorithm for this procedure is presented that efficiently solves large-scale problems. Solution examples with up to sixty-six thousand variables are described.

ESTIMATION OF AN ORIGIN-DESTINATION TRIP TABLE BASED ON OBSERVED LINK VOLUMES AND TURNING MOVEMENTS. VOLUME 1: TECHNICAL REPORT

Gur, Y.J., M. Turnquist, M. Schneider, L. Leblanc, and D. Kurth (John Hamburg and Associates, Inc., Rockville, MD). Sponsored by the Federal Highway Administration, Washington, DC. Report No. FHWA/RD-80/034 (May 1980): 119.

The LINKOD modelling system estimates a trip table based on observed link volumes and turning movements. This trip table can be used as an input to traffic assignment and/or simulation models. LINKOD is designed for small area analysis where all link volumes are known. The system is driven by the models: SMALD, a trip distribution model for small areas, and ODLINK, an adaptable trip table assignment model. SMALD estimates a trip table (the Target table) based on trip rates at sources and sinks and the network's service level. The ODLINK model corrects an input (Target) trip tables, so it best replicates the observed flows when assigned using equilibrium assignment. The models are driven by the LINKOD software package. The package includes several computer programs which perform input data editing and extensive summaries of the results.

ESTIMATION OF AN ORIGIN-DESTINATION TRIP TABLE BASED ON OBSERVED LINK VOLUMES AND TURNING MOVEMENTS. EXECUTIVE SUMMARY

Gur, Y.J. (Hamburg (John) and Associates, Inc., Rockville, MD). Report No. FHWA/RD-80/037 (July 1980): 11.

The objective of the project was to develop a model for estimating an origin-destination (O-D) trip table for highway traffic in small areas based primarily on observed link volumes and

turning movements. The major use of the trip table would be as an input to traffic assignment and simulation models for estimation of network flows and performance. The model developed in the project, the LINKOD model, has achieved its stated objective. This represents a significant advance in traffic modelling. There are three major features: (1) An existing trip distribution model has been enhanced to account for the unique attributes of small area O-D movements; (2) A technique for correcting an input trip table so it best reflects observed link volumes has been implemented. This technique is based on the theory of equilibrium traffic assignment; and (3) The model is driven by a software package which permits efficient applications.

THE APPLICATION OF DECOMPOSITION TO TRANSPORTATION NETWORK ANALYSIS

Dantzig, G.B., S.F. Maier, and Z.F. Lansdowne (Control Analysis Corp., Palo Alto, CA). Sponsored by the Transportation Systems Center, Cambridge, MA and the Department of Transportation, Washington, D.C., Office of the Secretary. Report No. DOT-TSC-OST-76-26 (October 1976): 127.

This document reports preliminary results of five potential applications of the decomposition techniques from mathematical programming to transportation network problems. The five application areas are (1) the traffic assignment problem with fixed demands, (2) the traffic assignment problem with elastic demand, (3) the transportation network improvement problem, (4) the optimal staging of transportation investments over time, and (5) the geographic decomposition of the traffic assignment problem. For all five, proposed solution techniques are presented and compared with previous work.

DEVELOPMENT OF A PEAK PERIOD TRAFFIC ASSIGNMENT CAPABILITY

Benson, J.D., C.E. Bell, and V.G. Stover (Texas Transportation Institute, Texas A&M University, College Station, TX; Texas State Department of Highways & Public Transportation, Transportation Planning Division, Austin, TX; Federal Highway Administration, Washington, D.C.). Report No. FHWA/TX-89/454-1F (August 1988): 102, Fig., Tab. 3.

The basic objective of this study was to develop and incorporate into the Texas Travel Demand a peak hour or peak period travel demand modelling capability. Peak hour and peak period travel demand modelling techniques vary considerably in their level of sophistication. These techniques can generally be categorized into three basic approaches: (1) factoring of 24-hour trip tables, (2) factoring of 24-hour trip ends and (3) direct generation. Two sets of data analyses were performed: (1) analyses of traffic count data from 254 locations in Houston and (2) analyses of peak period data from the recent Houston travel survey. Based on the results of these analyses and some basic conceptual concerns, the use of three-hour peak periods instead of a single peak hour for travel demand modelling applications is strongly encouraged. Perhaps the most important product of this

study is the software. Three new routines were developed, tested and implemented in the Texas Travel Demand Package to provide for peak period modelling applications.

RETURN TO THE PEAK? - TRANSPORTATION PLANNING METHODS.

Kroes, E.P., R.W. Antonisse, and S. Bexelius. Seminar C (Bath University, England). PTRC Education and Research Services Ltd. Report No. P290 (September 1987): 233-45.

The paper gives a short overview of the existing literature on the effects of congestion on choice of time of travel and presents and discusses methods to predict these effects. The paper presents the results of two studies looking at the effects of congestion on choice of time of travel. First a short overview will be given of available evidence, including empirical findings and some theoretical analyses. Then we look at methods of predicting responses in choice of time of travel to existing and future bottleneck situations. We distinguish between two types of methods: (1) rule of thumb methods, predicting arrival time pattern changes at individual bottlenecks from sparse information and (2) comprehensive methods, allowing network-wide forecasts of changes in choice of time travel, including possible interactions between multiple bottlenecks, and their effects on assigned traffic volumes. The methods are presented in some detail, and their applicability and approximate accuracy will be discussed. All methods are illustrated by a practical example where they are applied to a real-life case of capacity expansion which has been monitored in a before-and-after study. The predicted time of travel changes are compared with actual changes observed. For the covering abstract of the seminar see IRRD 816404. (Author)

IMPROVEMENT TO TRAFFIC ASSIGNMENT TECHNIQUES IN EXISTING SOFTWARE

Allen, D.W. (Institute of Transportation Engineers 56th Annual Meeting, Indianapolis, IN). ITE Compendium of Technical Papers. (September 1986): 176-179.

The past decade has seen considerable growth and development available to traffic engineers and planners. For traffic assignment and network evaluation, two trends have dominated. First, large mainframe-based programs have been developed and enhanced for more flexible or more accurate simulation of traffic operations. Second, microcomputer-based software has been developed by consultants, local and state agencies, and universities for planning and design applications. This paper presents possible approaches to resolving two recurring problems in practical application of microcomputer techniques: (1) inaccurate assignments on local streets and (2) overassignment on congested facilities. While the ideas may also be of interest to developers and users of mainframe-based systems, they are intended for practitioners developing or working with existing microcomputer techniques under budget restraints.

DERIVATION OF A SPATIALLY CONTINUOUS TRANSPORTATION MODEL

Tobler, W. (California University, Santa Barbara, CA). Transportation Research. Part A: General 19A, No. 2 (March 1985): 169-172.

A geographically continuous movement model can be deduced from the quadratic transshipment problem by considering the transportation network as a discrete mesh. The resulting system of coupled Helmholtz equations can be considered to solve simultaneously the spatially continuous versions of the traffic "distribution" and "assignment" problems. The associated Lagrangian functions are similar to the potentials of classical flow theory. Numerical methods implement the procedure. (Author)

TRANSPORTATION AND TRAFFIC THEORY

Gartner, N.H. and N.H.M. Wilson. The 10th International Symposium on Transportation and Traffic Theory (Massachusetts Institute of Technology, Cambridge, MA). (1987): 506.

The following papers were presented at the symposium: Performance of Urban Traffic Networks; Queue Evolution on Freeways Leading to a Single Core City During the Morning Peak; Analysis of the Effects of Parameter Estimation Error on Transportation Network Equilibrium Models; Traffic Control and Traffic Assignment in a Signal-Controlled Network with Queuing; A Theoretical Model to Calculate Speed Distribution as a Function of Density; Modelling and Study of Speed and Bunch Distributions Considering Fluctuations of Traffic Flow; Freeway Speed Distribution and Acceleration Noise - Calculations from a Stochastic Continuum Theory and Comparison with Measurements; Modelling and Filtering of Freeway Traffic Flow; Calculation of Signal Settings to Minimise Delay at Junction; Calculations of Optimum Fixed-Time Signal Programs; An Analysis of Traffic Performance of Major/Minor Priority Junctions with Non-Stationary Flow Demands; A Stochastic Environment for the Adaptive Control of Single Intersections; Simulation Study of OPAC: A Demand-Responsive Strategy for Traffic Signal Control; A Knowledge Based Expert System Architecture for Computer-Aided Analysis and Design of Intersections; Real-Time Identification of O-D Network Flows from Counts for Urban Traffic Control; Continued on TRIS Accession No 128xxx.

A VALIDATION OF THE SATURN AND ME2 MODELS USING BEFORE-AND-AFTER SURVEY DATA FROM MANCHESTER

Matzoros, T., D.V. Vliet, J. Randle, and B. Weston. Traffic Engineering and Control 28, No. 12 (December 1987): 641-643.

A series of tests was carried out using data collected in central Manchester before and after the introduction of a pedestrianization scheme involving road closures. The study was made to ascertain to what extent two models, SATURN and ME2, could be used to forecast correctly the impacts of the scheme. SATURN (Simulation and Assignment of Traffic to Urban Road Networks) is a general suite of programs for the analysis of road networks, including the capability to model in detail, delays at junctions. ME2 (Matrix Estimation

from Maximum Entropy) is one of a set of similar procedures by which the elements of a trip matrix can be estimated using data from roadside traffic counts. The details are given of the 'before' network and of the 'after' network, and of the modelled and observed travel times. The results of the study demonstrate that the SATURN and ME2 models taken together are capable of predicting the impacts of road closures on traffic flows with an accuracy of around twelve percent. Travel time predictions, while less accurate in absolute terms, were generally acceptable at the more aggregate levels.

CONSTRAINT QUALIFICATION FOR A DYNAMIC TRAFFIC ASSIGNMENT MODEL
Carey, M. (Operations Research Society of America). Transportation Science 20, No. 1 (February 1986): 55-58.

This note resolves an open question as to whether a dynamic traffic assignment model, which was developed and analyzed in earlier issues of this journal, satisfies a "constraint qualification." It is shown that the model does, in fact, satisfy a constraint qualification which establishes the validity of the optimality analysis presented by previous authors. (Author)

TRANSPORTATION PLANNING MODELS - WHAT THE PAPERS SAY

Atkins, S.T. Traffic Engineering and Control 27, No. 9 (September 1986): 460-467, Tab. 1.

Papers on transportation planning models are reviewed, and observations and statements made by academics and practitioners about transportation planning models are presented here. The review focuses on U.K. practice and references are made to the conventional four-stage modelling process. Academics and researchers regard the four-stage models as discredited. However, practitioners among local authorities and consultants often use these models. This paper examines reservations about their use. It reviews the following aspects of the modelling process: accuracy, structural deficiencies or specification errors of models, and the purpose of modelling exercises. Data collection, zoning and networks, trip generation, distribution models, modal split, assignment, calibration and validation, and forecasting ability are discussed. The purpose of modelling and special concerns of developing countries are also discussed.

INTERURBAN CAR DRIVER ROUTE CHOICE BEHAVIOR; A MODELLING APPROACH

Bergman, M.J., P. Barkey, and A.J. Daly (Dienst Verkeerskunde; Cambridge Systematics Europe Bv). Colloquium Vervoersplanologisch Spuurwerk, Delft, the Netherlands. 1 (1985): 23-52, Tab. 3.

The paper consists of two parts. Part One focuses on problems concerning the application of current assignment methods. A major issue, the calibration of assignment models, is

discussed. Furthermore, some behavioral considerations and some analytical objections are raised. This is the basis for the modelling approach described in the second part of the paper. Modelling is based on observed individual choices of route and uses a two stage approach. The choice set generation model is based on the principle that the large number of physical routes should be transformed into a smaller number of routes each representing a specific label. These labels have been defined as routes minimal with respect to one specific impedance criterion. Eventually, six such labels were selected which together match as large a number of observed choices as possible. From a behavioral point of view it seems plausible that for each individual only a small number of reasonable paths exists. The second stage involves a logit model representing the choice of one of these labelled routes on the basis of generic route attributes and attributes specific to a particular route or label. Market segmentation has been used to establish the importance of personal and trip characteristics. The estimation results demonstrate the importance of effect of factors other than time and distance. For the covering abstract of the conference see TRIS 453956. (Author/TRRL)

PRACTICAL AND THEORETICAL ASPECTS OF AGGREGATION PROBLEMS IN TRANSPORTATION PLANNING MODELS

Hearn, D.W. (Florida University, Gainesville, FL). Transportation Planning Models (International Center for Transportation Studies, Amalfi, Italy). (1984): 257-287, Fig. 17, Tab. 4.

Large scale traffic assignment models of urban transport networks are often prohibitively expensive computationally, prompting traffic engineers to use only subnetworks in the planning process. The author surveys three empirical investigations of this practice on networks for Phoenix, Arizona, Washington, DC, and Eindhoven, Netherlands. The extraction of subnetworks is then investigated theoretically and shown to have a rigorous basis in mathematical programming decomposition theory. Practical applications of the theory, which can improve the accuracy of current ad hoc procedures, are given with numerical examples. (TRRL)

ANALYTICAL TOOLS FOR TRANSPORTATION: A RESEARCH NOTE + TWO ARTICLES

Gomes, L.F.A. Departamento de Engenharia Industrial Pontificia Memorando Tecnico. No. 15 (1986).

Agosto de 1986 includes a note on the set of transportation impacts as a field and two articles: (1) modelling equilibrium traffic flow assignment with elastic demand as a stochastic nonlinear vector optimization problem and (2) operations research approach to the optimal control of parking systems.

ACCESSIBILITY, ENTROPY AND THE DISTRIBUTION AND ASSIGNMENT OF TRAFFIC REVISITED

Erlander, S. (Linköping Institute of Technology). Transportation Research. Part A: General 16B, No. 6 (December 1982): 471-472.

An optimizing model which minimizes average generalized trip cost subject to constraints on the entropy was given in a previous paper. In this note the model is placed in a planning context. (Author/TRRL)

TRAFFIC FORECASTING AT SCHEME LEVEL UTILIZING REGIONAL HIGHWAY TRAFFIC MODEL (RHTM) TRIP END DATA - TRANSPORTATION PLANNING PRACTICE

Findlay, C.M. (Jamieson Mackay & Partners). Seminar M (University of Warwick, England). PTRC Education and Research Services Ltd. Report No. P212 (1981): 81-91, Fig. 4, Tab. 4.

In the spring of 1978 Jamieson Mackay and Partners were commissioned to undertake the north east Clwyd traffic study. The objectives of the study were defined as: forecasting the likely long term traffic flows on the road network following the completion of major road improvements, and evaluating, from an operational and economic view point, solutions to any potential traffic problem areas. Survey data were collected to supplement the existing rhtm data. Traffic models were developed and calibrated for four separate vehicle purpose groups. The traffic models incorporated: zones and highway network, vehicle ownership, trip ends, trip distribution, trip assignment and behavioral parameters. The traffic forecasting stage of the study determined zonal growth factors for each of the vehicle purpose groupings. This paper concentrates on the creation of these growth factors from the trip end model output and their use for traffic forecasting purposes. The methodology is outlined. (TRRL)

THE EFFECT OF THE LEVEL OF SPATIAL DETAIL ON THE ESTIMATES OF INTER-ZONAL IMPEDANCES

Jansen, G.R.M. (Delft University of Technology, the Netherlands). Colloquium Vervoersplanologisch Speurwerk. (1981): 323-342, Fig. 7, Tab. 5.

An important source of error in transportation analysis results is the level of spatial detail applied (i.e., zone size and network detail). This level also greatly affects the analysis costs. In order to investigate these effects in the case of the assignment module, an experiment was designed allowing the systematic variation of factors like zone size, network detail and assignment model type. The experiment involves the application of a number of assignment models to three network models at varying levels of spatial detail. The city of Eindhoven (population = 200,000) is used for this purpose. The fine network model is identical with the complete road network. The coarse network model is rather skeletal. The medium

network is selected such that it has equal distances to both network models and may be more or less regarded as the level mostly used in European practice. This paper presents the results of an investigation into the sensitivity of impedances between zones, resulting from all-or-nothing assignments, to the level of spatial detail. Travel times, travel distances and airline distances were computed at three levels of detail for a large number of randomly selected pairs of locations within Eindhoven. Analysis showed that for each of these impedance measures the distributions at all levels of detail were nearly identical. Furthermore, the medium and coarse level impedances proved to be unbiased and fairly accurate estimates of the fine level impedances. In particular, the medium level values are accurate for the entire range of impedances. For the covering abstract of the conference see TRIS abstract number 367725. (TRRL)

JUNCTION INTERACTIONS AND MONOTONICITY IN TRAFFIC ASSIGNMENT

Smith, M.J. (York University, England). Transportation Research. Part B: Methodological 16B, No. 1 (February 1982): 1-3.

We consider the traffic equilibrium problem when the travel demand is inelastic and stationary in time. Junction interactions, which abound in urban road networks, are permitted. We prove that the set of equilibria (solutions to the assignment problem) is convex when certain monotonicity and continuity conditions are satisfied at each junction. (Author/TRRL)

TRANSPORTATION MODELS AND ANALYSIS

Heydecker, B.G., A.M. Budd, et al. Seminar P (University of Warwick, England). PTRC Education and Research Services Ltd. Report No. P198 (1980): 382.

The proceedings of Seminar P contain the following papers: Some Theoretical Difficulties in Using Equilibrium Assignment in Signal-Controlled Road Networks (Heydecker, B.G.); Modelling Dynamic Junction Behavior (Budd, A.M. and Mackenzie, J.); The Effect of Spatial Detail on Equilibrium Assignment Results (Jansen, G.R.M. and Bovy, P.H.L.); Development and Application of a Park-and-Ride Model (le Clercq, F.); Research on Household Car Ownership (Bates, J.J. Roberts, M.); A Behavioral Model of Household Ownership and Use of Motor Vehicles and its Application for Aggregate Forecasting (Ben-Akiva, M., Manski, C.F. and Sherman, L.); Computing in Transport Planning in the 1980's: A U.S. Perspective (Manheim, M., Gonzalez, S., Litinas, N. and Salomon, I.); Computing in Transport Planning in the 1980's: A British Perspective (Wootton, H. J.); RHTM Trip Distribution Investigation (Kirby, H.R., Gunn, H.F., Murchland, J.D. and Whittaker, J.C.); RHTM Base Year and Future Year Trip Matrices (Case, D.J. and Catling, I.); Uncertainty-the Errors of the Forecast of an Inter-Urban Traffic Model (Ashley, D.J. and Shewey, P.); An Analysis of Airport Passenger Surface Access Mode Choice at a Major British Provincial Airport (Sheldon, R.J.). (Continued on TRIS 342936).

NOTE ON THE ACCURACY OF THE CONTINUUM APPROXIMATION SPATIAL AGGREGATION ALGORITHM OF TRAFFIC ASSIGNMENT

Sheffi, Y. and R. Trexler (Massachusetts Institute of Technology, Cambridge, MA). Operations Research Society of America. Transportation Science 14, No. 4 (November 1980): 306-323.

This paper investigates the accuracy of the probit approximation based solution of the spatial aggregation problem of traffic assignment. It tests the normal approximation to uniform intrazonal travel times and the C. Clark approximation to the solution of the choice probabilities between alternatives characterized by normally distributed travel times. Some guidelines for using the technique are developed.

THE QUADRATIC ASSIGNMENT PROBLEM: AN ANALYSIS OF APPLICATIONS AND SOLUTION STRATEGIES

Liggett, R.S. (California University, Los Angeles, CA). Environment and Planning B 7, No. 2 (1980): 141-162, Fig. 5, Tab. 5.

A wide variety of practical problems in design, planning and management can be formulated as quadratic assignment problems, and this paper discusses this class of problem. Since algorithms for producing optimal solutions to such problems are computationally infeasible for all but small problems of this type, heuristic techniques must usually be employed for the solution of real practical problems. This paper explores and compares a variety of solution techniques found in the literature, considering the trade-offs between computational efficiency and quality of solutions generated. Recommendations are made about the key factors to be considered in developing and applying heuristic solution procedures. (Author/TRRL)

OPTIMAL TRAFFIC ASSIGNMENT WITH ELASTIC DEMANDS: A REVIEW-ANALYSIS FRAMEWORK

Gartner, N.H. (Lowell University). Operations Research Society of America. Transportation Science 14, No. 2 (May 1980): 174-191.

This study reviews, in two parts, the formulation, interpretation, and solution methodology of the traffic assignment problem with elastic demands, originally described by M. I. Beckmann, C. B. McGuire and C. B. Winsten. This paper defines the different possible modes of traffic assignment in a network and identifies the economic rationales of the associated extremum formulations.

OPTIMAL TRAFFIC ASSIGNMENT WITH ELASTIC DEMANDS: A REVIEW - 2. ALGORITHMIC APPROACHES

Gartner, N.H. (Lowell University). Operations Research Society of America. Transportation

Science 14, No. 2 (May 1980): 192-208.

The paper examines algorithmic approaches for calculating the flow patterns resulting from the different modes of assignment. An efficient methodology for solving the elastic-demand TAP is based on remodelling it as an equivalent assignment problem in an expanded network. The variable-demand TAP is then transformed into a fixed-demand TAP with a trip table consisting of the potential demands and can be solved by available fixed-demand assignment algorithms. Three alternative transformations are described.

NETWORK REPRESENTATION, CONTINUUM APPROXIMATIONS AND A SOLUTION TO THE SPATIAL AGGREGATION PROBLEM OF TRAFFIC ASSIGNMENT

Daganzo, C.F. (California University, Berkeley, CA). Transportation Research. Part B: Methodological 14B, No. 3 (September 1980): 229-239, Fig. 8, Tab. 1.

This paper complements the preceding one which showed how one could modify equilibrium traffic assignment algorithms for networks with many centroids. In this one it is shown how one can substitute centroids by zones with continuous population densities. The technique, which is mathematically guaranteed to approximate a flow pattern in accordance with Wardrop's user equilibration criterion, requires some geometrical calculations which can be handled off-line. With the suggested approach, it is possible to represent the spatial distribution of trip ends more realistically and this allows traffic assignment models to capture the effects of short and intrazonal trips in a way previously impossible. The method is shown to be computationally feasible. (Author/TRRL)

AGGREGATION IN TRANSPORTATION PLANNING: NETWORKS, ZONES AND TRANSPORTATION MODELS

Martin Center for Architect & Urban Studies Trans N4 (1980): 207-229.

If we consider traffic networks, zones, transportation models and their associated interactions as a system, then aggregation can be viewed as a technique for simplifying this system, subject to certain consistency requirements. In this paper, a number of invariance criteria are defined and tested. A series of aggregations along links, routes and zones are incorporated within a combined trip-distribution/traffic-assignment model. The solution to the various problems of aggregation discussed in this paper is shown to lie in the use of a generalized cost function with appropriate weights which is developed in a form which provides a one-to-one correspondence with each aggregation step. (Author/TRRL)

APPLIED NETWORK OPTIMIZATION

Mandl, C. (1980): 176.

This is a detailed treatment of network optimization problems in transport and urban and

regional planning. Most of the book is devoted to the problems of network design for road, rail and air transport, pipeline and waste water systems and urban public transport. More specifically, network flow problems like traffic assignment and vehicle routing for school buses and waste collection are discussed. (Pergamon Press)

PRESENT AND FUTURE CONSIDERATIONS OF INTERACTIONS BETWEEN AUTOMOBILES AND TRAFFIC CONTROL SYSTEMS

Institute of Transportation Studies. The International Symposium on Traffic Control Systems (California University, Berkeley, CA). Federal Highway Administration, Washington D.C. Analysis and Evaluation 2D (December 1979): 278.

Contents: Measures of effectiveness for urban traffic management; Capacity and quality of flow on urban arterials; Traffic data collection system for the Belgian motorway network measures of effectiveness aspects; Effectiveness of freeway traffic control systems with respect to environmental protection; Simulation and assignment of traffic in urban road networks; A microscopic discrete-event time-oriented traffic simulation model for a two-lane highway network; Assessment of integrated traffic control systems in downtown areas by a complex simulation model; A simulation model for signalized intersections; Computer-aided design and evaluation of traffic systems; Scope and limitations of analytical models in optimizing traffic control in urban streets; Development of a dynamic traffic assignment model of urban street networks; Route choice and traffic control in central urban areas; The comprehensive automobile traffic control system - a look at the future; Interactions between automobiles and traffic control systems; On-board logic for interaction with traffic control systems; and criteria for planning interactive traffic control systems. See also Volume 2C, PB80-180706. Sponsored in part by Federal Highway Administration, Washington, D.C. Also published as ISSN-0192-4117.

A DISCRETE-CONVEX PROGRAMMING APPROACH TO THE SIMULTANEOUS OPTIMIZATION OF LAND USE AND TRANSPORTATION

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Three design problems are discussed in this article. First, it is shown that the transportation network design problem with congestion reduces to an all-or-nothing traffic assignment problem under some assumptions on the congestion function and the investment cost function. Second, the land use design problem is formulated as an extension of the Xoopmans-Beckmann problem, and a heuristic is proposed to solve this problem. Third, it is shown that the seemingly more complex problem of designing jointly a land-use plan and a transportation network reduces to a pure land-use design problem. All that is needed to solve the joint optimization problem is a shortest path algorithm and a heuristic to solve the land use design problem. Computational experience is reported for each algorithm. (TRRL)

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