SOME TRAFFIC SYSTEM ANALYSIS TECHNIQUES

by

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One of a Series on
National Cooperative Highway Research Program
Project 20-3

"OPTIMIZING FREEWAY CORRIDOR OPERATIONS THROUGH TRAFFIC SURVEILLANCE, COMMUNICATIONS AND CONTROL"

Texas Transportation Institute
Texas A&M University
College Station, Texas

September, 1967

ACKNOWLEDGMENT

This work was sponsored by the American Association of State Highway Officials, in cooperation with the Bureau of Public Roads, and was conducted in the National Cooperative Highway Research Program which is administered by the Highway Research Board of the National Academy of Sciences--National Research Council. Publication of the work does not necessarily indicate acceptance by the Academy, the Bureau of Public Roads or by any State Highway Department of the findings, conclusions or recommendations either inferred or specifically expressed therein.

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PREFACE

This report is the fourth in a series of reports on National Cooperative Highway Research Program Project 20-3, "Optimizing Freeway Corridor Operations Through Traffic Surveillance, Communication and Control." To date the series includes:

Report 488-1 "Interim Report on the Evaluation Phase" by Joseph A. Wattleworth, Charles E. Wallace, Moshe Levin and J. G. Sample.

Report 488-2 "Evaluation of the Operational Effects of an On-Freeway Control System" by Joseph A. Wattleworth and Charles E. Wallace.

Report 488-3 "Development and Evaluation of a Freeway Ramp Control System on the Lodge Freeway" by Joseph A. Wattleworth, Charles E. Wallace and Moshe Levin.

Report 488-4 "Some Traffic System Analysis Techniques" by Joseph A. Wattleworth and Charles E. Wallace.

One of the major activities of NCHRP Project 20-3 was the development and evaluation of a ramp metering system on the northbound Lodge Freeway. In order to evaluate completely the effects of the ramp control system, extensive analyses of travel on the arterial streets in the Freeway Corridor had to be made in addition to those on the Freeway. These analyses needed a firm statistical base and the measures of effectiveness of operation in the Corridor had to be the same as those on the Freeway. The analysis techniques and statistical models reported herein were developed so that a total systems analysis could be made.

INTRODUCTION

NEED FOR NETWORK ANALYSIS TECHNIQUES

At the present time, one of the biggest gaps in traffic technology is represented by the lack of an adequate procedure for evaluating in a quantitative manner the operation of a large traffic system - a street network with or without a freeway. Most present study techniques are primarily point studies (counts, intersection studies) or simply moving vehicle (travel time or acceleration noise) studies and yield very little data of a system type.

This type of traffic system analysis procedure is needed to evaluate quantitatively the effect of operational changes within a network. The operational changes might involve changes in network signal timing, change to one-way street operation, reversible-lane operation on a street or a myriad of other possibilities. The traffic system analysis techniques reported herein were developed to evaluate the effects of freeway ramp metering control systems on network operation in Houston and Detroit.

Figure 1 shows the John C. Lodge Freeway Area of Influence in the afternoon peak period. It is this system in which the Texas Transportation Institute is currently evaluating the effects of a system of freeway ramp metering controls.

REQUIREMENTS OF THE ANALYSIS TECHNIQUES

Some of the requirements which must be met by the network

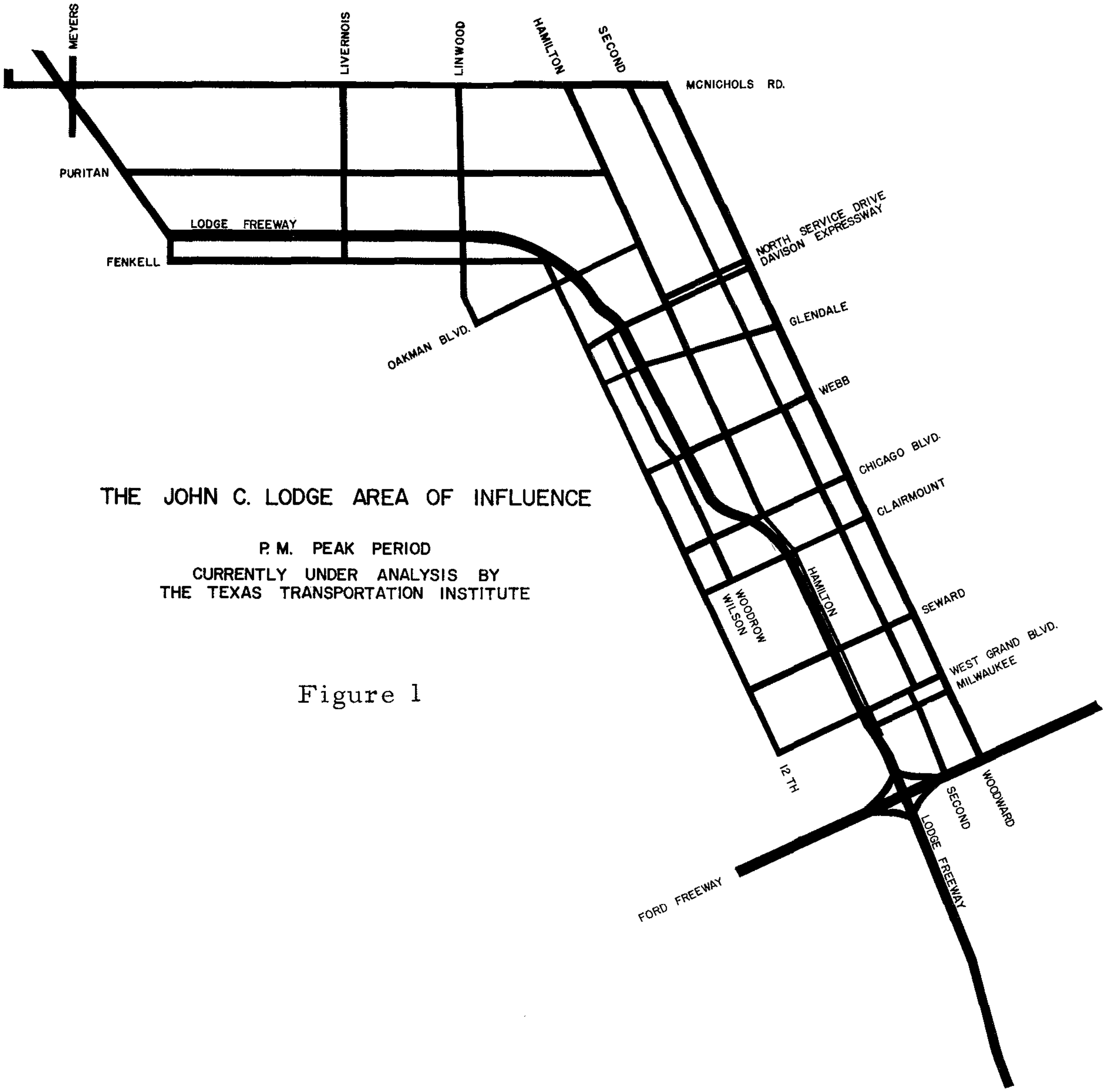

MEYERS
LIVERNOIS
LINWOOD
HAMILTON
SECOND
MCNICHOLS RD.
PURITAN
LODGE FREEWAY
FENKELL
NORTH SERVICE DRIVE
DAVISON EXPRESSWAY
OAKMAN BLVD.
GLENDALE
WEBB
CHICAGO BLVD.
CLAIRMOUNT
HAMILTON
WOODROW WILSON
SEWARD
WEST GRAND BLVD.
MILWAUKEE
12 TH
SECOND
WOODWARD
LODGE FREEWAY
FORD FREEWAY
THE JOHN C. LODGE AREA OF INFLUENCE
P. M. PEAK PERIOD
CURRENTLY UNDER ANALYSIS BY
THE TEXAS TRANSPORTATION INSTITUTE

Figure 1

analysis techniques are:

1. they must be capable of use in the analysis of large networks,

2. they must be capable of use in the analysis of networks which include both freeways and/or surface streets,

3. they must be capable of providing system measures,

4. the measurements and measures of effectiveness obtained on the freeway must be consistent with those obtained on the streets,

5. they must provide measurements which are sensitive enough to reflect changes in system operation, and

6. they must not exceed reasonable cost limits.

REPORT ORGANIZATION

Six major sections of the report follow. The first of these deals with general system considerations including the measures of effectiveness used in evaluating network operations. Next is a section which describes the input-output technique of obtaining these measures of effectiveness on freeways.

Next is a section which presents the techniques used to obtain the same measures of effectiveness on a system of streets. The means of analyzing and evaluating operational changes on a single route is described in a following chapter.

Implementation of the network analysis procedures described requires a great deal of data processing and coordination and a network information system is presented. Finally the conclusions are presented.

GENERAL SYSTEM CONSIDERATIONS

In the analysis of a large traffic system it is necessary that true system measures are used. Many of the more commonly used traffic engineering measures are not system measures-for example volume and density are not system measures. That is, they are not additive from one subsystem to another or from one time period to another or simply do not reflect system operation.

Several measures which are system measures are as follows:

1. number of vehicles in a system or subsystem
2. total travel time
3. total travel
4. average speed (derived)
5. kinetic energy

Various means of obtaining these measures are presented in the paper.

The function of number of vehicles on each link ℓ versus time can be integrated to yield, for any time period, the total travel time in vehicle-hours (TTT_ℓ)(References 1 and 2). The volume count q_ℓ on the link for the same time period multiplied by the link length d_ℓ yields the total travel (TT_ℓ) on the link in terms of vehicle-miles. The volume count, q_ℓ, on the link multiplied by the average travel time, t_ℓ, on the link can also give the total travel time on the link. From these data, the average speed on the link during the time period can be calculated and is $TT_\ell / TTT_\ell = \mu_\ell$. The kinetic energy (3, 4) of the traffic stream on the link can also be calculated and is

$KE_{\ell} = d_{\ell} q_{\ell} u_{\ell} = d_{\ell} q_{\ell} \ (TT_{\ell} / TTT_{\ell}) = d_{\ell} (1/d_{\ell}) \ (TT_{\ell}^{2} / TTT_{\ell}) =$

$(TT_{\ell}^{2} / TTT_{\ell})$. Kinetic energy has proved to be quite a useful description of the quality of traffic flow or level of service as are the total travel time, total travel and average speed.

Total travel time (when used over proper time periods and for the proper system) represents a true system measure and figure of merit. It is particularly useful in systems analysis work for two reasons: 1) for any time period, T_{12} the total travel time in two (mutually exclusive) subsystems A and B is the sum of the total travel time in subsystem A and the total travel time in subsystem B and 2) for any subsystem the total travel time during two (mutually exclusive) time periods T_{12} and T_{23} is equal to the sum of the total travel time during period T_{12} and the total travel time during the period T_{23}.

To summarize:

1. $(TTT)_{A+B}^{1-2} = (TTT)_{A}^{1-2} + (TTT)_{B}^{1-2}$

2. $(TTT)_{A}^{1-2+2-3} = (TTT)_{A}^{1-2} + (TTT)_{A}^{2-3}$

where $(TTT)_{A}^{1-2}$ is the total travel time in subsystem A during time period 1-2

and, where subsystems A and B are mutually exclusive and time periods T_{1-2} and T_{2-3} are mutually exclusive.

Similarly, the total amount of travel in a system (vehicle miles) is a system measure and the following relationships hold.

1. $(TT)^{1-2}_{A+B} = (TT)^{1-2}_{A} + (TT)^{1-2}$

2. $(TT)^{1-2+2-3}_{A} = (TT)^{1-2}_{A} + (TT)^{2-3}_{A}$

where $(TT)^{1-2}_{A}$ is the total travel in subsystem A during time period 1-2

and where subsystems A and B are mutually exclusive and time periods T_{1-2} and T_{2-3} are mutually exclusive.

If properly used, average speed is also a useful system measurement. If the average speed in subsystem A in time 1-2 $U^{1-2}_{A} = TT^{1-2}/TTT^{1-2}_{A}$ and if the average speed in subsystem B is $U^{1-2}_{B} = TT^{1-2}_{B}/TTT^{1-2}_{B}$, the following relationships hold:

1. $U^{1-2}_{A+B} = (TT^{1-2}_{A} + TT^{1-2}_{B})/(TTT^{1-2}_{A} + TTT^{1-2}_{B})$

2. $U^{1-2+2-3}_{A} = (TT^{1-2}_{A} + TT^{2-3}_{A})/(TTT^{1-2}_{A} + TTT^{2-3})$

where subsystems A and B are mutually exclusive and time periods T_{1-2} and T_{2-3} are likewise mutually exclusive.

Of course, if these same measurements are available for an entire network, it is possible to determine these same parameters of the network. For example if TT_{ℓ} and TTT_{ℓ} are the total travel on link ℓ

and the total travel time on link ℓ , respectively, the total travel in the entire system is $TT_s = \Sigma TT_\ell$ and the total travel time in the system is $TT_s = \sum_\ell TTT_\ell$. Thus, during the time period under consideration the average speed in the system is $\mu_s = TT_s / TTT_s$. The kinetic energy in the system is $KE_s = (TT_s^2 / TTT_s)$.

FREEWAY SYSTEM ANALYSIS - THE INPUT-OUTPUT TECHNIQUE

The analysis of freeway operation has been accomplished quite successfully through the use of the input-output technique (5, 6, 7). This analysis technique was developed by the Texas Transportation Institute on the Houston Freeway Surveillance and Control Project (5), and was later applied more extensively in Chicago (6). Only in the later application of the Technique on the Lodge Freeway in Detroit by TTI (7) have its real capabilities been approached. This chapter presents some further refinements of the technique.

Let us assume that the system of interest is the straight-pipe freeway section, one mile in length, shown in Figure 2. The study consists of coordinated counts at both ends of the section to determine the input-output characteristics of the section. In order to start the study a marked vehicle is driven through the section. At time t_o - 1 minute the vehicle arrives at the input line and the input count of V_{in} is started. Assuming a one minute travel time between the stations and that no vehicles passed the signal car or were passed by it, the vehicle arrives at the output line at time t_o and the output count V_{out} is started.

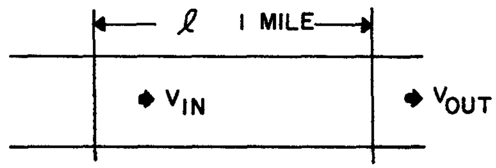


Figure 2. Simple Straight-Pipe Freeway Section

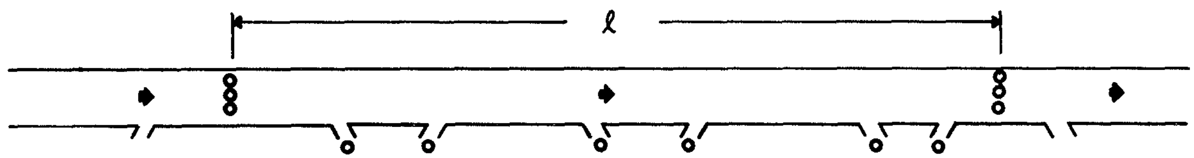


Figure 3. Freeway System Showing Count Stations for Input-Output Study

If N_o is the original number of vehicles in the system at time t_o, $N_o = \left[V_{in}(t_o) - V_{out}(t_o)\right] = V_{in}(t_o)$. Therefore, if N(t) is the number of vehicles within the section at time t, $N(t) = \left[V_{in}(t) - V_{out}(t)\right]$.

If all data are recorded simultaneously at five minute intervals, the following information are available during any five minute period [t, t+5] .

1. The input volume during the five minute period $\left[V_{in}(t+5) - V_{in}(t)\right]$.
2. The output volume during the period $\left[V_{out}(t+5) - V_{out}(t)\right]$
3. The number of vehicles in the section at time t, N(t)
4. The number of vehicles in the section at time t+5, N(t+5).

Let us examine a simple set of data for a five minute period, say from 4:30 to 4:35 PM. Let us assume the following data:

(a) N (4:30) = 100 vehicles

(b) V_{in} (4:30-4:35) = 400 vehicles

(c) V_{out} (4:30-4:35) = 410 vehicles

We can from this immediately state that N (4:35) = 100 + (400-410) = 90 vehicles.

The average flow over the section is 1/2 (400 + 410) or 405 vehicles per 5 minutes. Therefore the total travel within the section from 4:30 to 4:35 is 405 vehicles x 1 mile = 405 vehicle miles. The hourly rate of travel within the section is 12 x 405 = 4860 vehicle miles per hour.

The total travel time within the section from 4:30 to 4:35 can be obtained by simple integration

= 1/12 x 1/2 x (100 + 90) = 7.9 vehicle hours.

The average speed within the section is the total travel, TT, divided by the total travel time, TTT

= (TT)/(TTT) or 405 vehicle hours/7.9 vehicles hours

= 51.2 miles per hour.

The total kinetic energy of a traffic stream was determined by Drew (3) to be

KE = qu where q is volume in vehicles per hour and u is the speed in miles per hour.

This quantity represents energy per unit length of highway per unit time. Hence, to determine the energy in a section it would be necessary to multiply the volume times speed times section length

or KE = qul.

This assumes a constant q over the entire section. If this assumption is true, ql represents the total travel, TT, per hour within the section. Thus, the total kinetic energy of a section is the total travel, TT, (vehicle miles) times the average speed. Since the average speed = (TT)/(TTT) the kinetic energy of the traffic stream = $(TT^2/(TTT)$ for the period of interest.

The kinetic energy of the traffic stream within the section during the 4:30-4:45 period is

KE (4:30-4:35) = (405 veh mi) x (51.2 mph) x (1/mi-hr)

= $(405 \text{ veh mi})^2/(7.9 \text{ veh hrs}) \times (1/\text{mi-hr})$

= $20,736 \text{ veh mi/hr}^2$.

The rate accumulation of energy in the section per hour

$$= 12(20,736) = 248,832 \text{ veh mi/hour}^2.$$

The point of this discussion is to show the large amount of information--truly system information--which can be obtained from a series of point studies. Point studies are the easiest to make and are therefore desirable from the data collection point of view. Much more important, however, is the fact that the data obtained are the proper type. For example, from the input-output studies several excellent figures of merit of system operation are obtained, namely, total travel time, total travel, average speed and kinetic energy.

Figure 3 shows a system which can exemplify the procedure. With the count stations as shown, the system would be the freeway within the count points--or that area cordoned off by the count stations. Within this area all of the following data could be obtained for any time period.

1. freeway input volume
2. freeway output volume
3. total ramp input volume
4. total ramp output volume
5. total system travel time
6. total system travel
7. average speed in the system
8. kinetic energy in the system

NETWORK ANALYSIS TECHNIQUES

The analysis techniques presented here are applicable for several purposes. However, the primary utility is in the analysis of operation in the network under two control or operational conditions where fairly long time periods are available for observations under each condition. An example of this is the evaluation of the effect of the implementation of a freeway ramp metering control system - the purpose for which the techniques were developed. Fairly long time periods, several weeks or more, are available for data collection under each of the two operational conditions.

Even with these long time periods for data collection, sampling procedures will normally be required in large networks and later sections present statistical models to aid in the design of the sampling procedure and the sampling procedures themselves are also presented.

In Detroit a stratified sampling procedure is used by TTI for the analysis of the John C. Lodge Freeway corridor. In this corridor (see Figure 1) the north-south movement is the predominant movement and the east-west flow is relatively minor. Therefore, the sampling frequency of the north-south streets is higher than on the east-west streets.

As described in Section II, GENERAL SYSTEM CONSIDERATIONS, total travel and total travel time on a link or in the system are the two prime measurements. When these two are known, average speed and kinetic energy on the link as in the system can be calculated.

METHOD OF ESTIMATING TOTAL TRAVEL IN A SYSTEM

The total travel must be determined for the same network for which total travel time is estimated. The basic information which is required is the length of each link in the system and the volume count on each link in each time period. For small systems it would merely be necessary to obtain continuous volume count data on each link during the study period by machine or manual methods.

The problem is different, however, in a large network. In this case simultaneous counting of all links is probably not feasible so some sort of sampling procedure is appropriate. Since the purpose of the entire system technique is to compare traffic operation in a network under different states of operation or control, long periods of comparison are required. Average values of total travel time and total travel will be determined during the two comparison periods (probably by sub time periods and by small sections so operational comparison of subsystems can be made).

SAMPLING TECHNIQUES

Basically two types of volume sampling procedures could be used. The first would involve the continuous counting of traffic on a given set of links on a given day and the design of the sampling program would be the selection of the links to be counted on each day of the study. Thus for n counters (either manual or machine) n locations would be counted. A machine count program in a large system would require changing of the locations of most of the count stations frequently and hence would likely be prohibitive. Other problems, such as count inaccuracy and counter failures, precluded the extensive use of this method for the purpose of the type of research for which it was developed.

The sampling procedure designed by the Texas Transportation Institute for use in Detroit on NCHRP Project 20-3 is based on the expansion of six minute counts to estimate hour volumes. This basic technique has been described by the National Committee on Urban Transportation and evaluated by Gilbert (8) who found it to be about as accurate as machine count techniques for most volume ranges.

The counts are made manually once per hour at each location. Each man can count at six to eight locations each hour and returns to count the same locations each hour at the same minute period. Table 1 shows a sample count schedule for one man in the John Lodge Freeway corridor in Detroit. If each man counts 8 links in a day and if n men are available, volume samples would be made on 8n links in any one day.

STATISTICAL MODEL

From the count samples on the links, it is necessary to estimate the average total travel (vehicle-miles) on a given link and in the system in any time period (such as 4 to 5 p.m.).

Let us assume that the 4-5 p.m. time period is of interest and that the average total travel is being estimated for a period of N days. In other words we want to determine the average travel on individual links and in the entire system for the N-day period during some time period T (such as 4-5 p.m.).

For a given link, ℓ, let us assume that a six-minute sample count was taken during time period T on the link of n_ℓ days during the N day period. Let $q^T_{i,\ell}$ be the six minute volume count on link ℓ on day i during the time period T. Then the best estimate of the volume $Q^T_{i,\ell}$ during the hour period T is $10\, q^T_{i,\ell}$ and, if d_ℓ is the length of link ℓ in miles, the best estimate of total travel on link ℓ on day i during time periods is $TT^T_{i,\ell} = d_\ell\, Q^T_{i,\ell} = 10 d_\ell\, q^T_{i,\ell}$.

Now considering all of the n_ℓ samples of volume on link ℓ in period T, the best estimate of the average total travel in time period T on link ℓ is

$$TT^T_\ell = \frac{(10\, d_\ell)}{n_\ell} \left(\sum_{i=1}^{n_\ell} q^T_{i,\ell} \right).$$

The average amount of total travel in the entire system TT^T_s is merely the sum of the average of travel on each link (TT^T_ℓ) for all L links. Thus the average total travel in the system during time period T

$$TT^T_s = \sum_{\ell=1}^{L} (TT^T_\ell) = \sum_{\ell=1}^{L} \left(\frac{10 d_\ell}{n_\ell} \sum_{i=1}^{n_\ell} q^T_{i,\ell} \right)$$

The variance of volume and total travel on a link and of total

TABLE 1. POSSIBLE SAMPLING COUNT SCHEDULE - DETROIT AREA

Count Station Number	Count Station Location	Links Counted	Time of Sample			
			Hour 1	Hour 2	Hour 3	Hour 4
1	Woodward at Grand	250-233, 231-237	3:00-06	4:00-06	5:00-06	6:00-06
2	Woodward at Seward	233-212, 211-212	3:10-16	4:10-16	5:10-16	6:00-16
3	Woodward at Clairmount	212-203, 202-203	3:20-26	4:20-26	5:20-26	6:20-26
4	Second at Clairmount	200-202, 211-202	3:30-36	4:30-36	5:30-36	6:30-36
5	Second at Seward	209-211, 231-211	3:40-46	4:40-46	5:40-46	6:40-46
6	Second at Grand	229-231, 248-231	3:50-56	4:50-56	5:50-56	6:50-56

travel on a link and of total travel in the system can be derived from the well-known properties of variances of random variables (9). For example, if random variable x has a mean μ_x and a variance σ_x^2 and if random variable y has a mean μ_y and a variance σ_y^2,

1. The random variable x + y has

   a. a mean $\mu_{x+y} = \mu_x + \mu_y$

   b. a variance $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$

2. The random variable ax (where a is a constant) has

   a. a mean $\mu_{ax} = a\mu_x$

   b. a variance $\sigma_{ax}^2 = a^2\sigma_x^2$

These relationships can be found in any good text on probability theory or statistics such as reference 9.

In the network analysis procedure, the basic random variable is $q_{i,\ell}^T$ which is the six minute volume sample count on link ℓ on day i during time period T. This is the random variable which is directly measured.

The variance of the six minute samples on link ℓ during time period T is simply

$$\text{Var } q_{i,\ell}^T = \left[\sum_{i=1}^{n_\ell} (q_{i,\ell}^T)^2 - \left(\frac{1}{n_\ell}\right) \left(\sum_{i=1}^{n_\ell} q_{i,\ell}^T \right)^2 \right] \Bigg/ n_\ell - 1$$

Since $Q_{i,\ell}^T = 10\ q_{i,\ell}^T$, the corresponding variance of the hour volume estimates on link ℓ during time period T is simply

$$\operatorname{var} Q_{i,\ell}^{T} = 10^{2} \operatorname{var} q_{i,\ell}^{T}$$

$$= \frac{100}{n_\ell - 1} \left[\sum_{i=1}^{n_\ell} (q_{i,\ell}^{T})^{2} - \left(\frac{1}{n_\ell}\right) \left(\sum_{i=1}^{n_\ell} q_{i,\ell}^{T}\right)^{2} \right]$$

Since the total travel on link ℓ on day i during time period T, $TT_{i,\ell}^{T} = d_\ell \, Q_{i,\ell}^{T}$, the variance of the total travel on link ℓ in time period T is

$$\operatorname{var} TT_{\ell}^{T} = d_\ell^{2} \operatorname{var} Q_{i,\ell}^{T} = 100\, d_\ell^{2} \operatorname{var} q_{i,\ell}^{T}$$

$$= \frac{100\, d_\ell^{2}}{n_\ell - 1} \left[\sum_{i=1}^{n_\ell} (q_{i,\ell}^{T})^{2} - \left(\frac{1}{n_\ell}\right) \left(\sum_{i=1}^{n_\ell} q_{i,\ell}^{T}\right)^{2} \right]$$

The total travel in a system during time period T equals the sum of the total travel on all of the links in the system during the same period T, or $TT_{s}^{T} = \sum_{\ell=1}^{L} TT_{\ell}^{T}$. Thus, the variance of the total travel in the system in time period T is

$$\operatorname{var} TT_{s}^{T} = \sum_{\ell=1}^{L} \operatorname{var} TT_{\ell}^{T}$$

$$= \frac{100}{n_\ell - 1} \sum_{\ell=1}^{L} \left\{ d_\ell^{2} \left[\sum_{i=1}^{n_\ell} (q_{i,\ell}^{T})^{2} - \left(\frac{1}{n_\ell}\right) \left(\sum_{i=1}^{n_\ell} q_{i,\ell}^{T}\right)^{2} \right] \right\}$$

A SAMPLING METHOD OF ESTIMATING TOTAL TRAVEL TIME IN A SYSTEM

The purpose of this is to describe an estimation procedure for determining the total travel time in a system during a particular time period during the day. The method is based on sampling procedures which, for large systems, will require several different days to obtain samples in all parts of the system. Hence, the total travel time in the system is an estimated average value over a period of perhaps several days or weeks. If the sampling frequency and/or duration can be increased, it is possible to estimate not only the mean but the variance of the total travel time in the system as well as on any link.

STATISTICAL MODEL

The estimation of total travel time in a large system like the estimation of total travel, is a massive sampling problem. For any time period, for example 4 to 5 p.m., the statistics of interest are the mean and variance (or standard deviation) of the total travel time in the system. This implies a period of several days of observations in order to obtain the variance, which is caused by daily variations in traffic and the sampling procedures used.

The mean and variance of total travel time in the system in time period T can be denoted by μ_s^T and $(\sigma_s^T)^2$, respectively and can be estimated by the statistics $(TTT)_s^T$ and var TTT_s^T which are the sample mean and variance, respectively, of the total system travel time. $TTT^T = \frac{1}{N} \sum_{i=1}^{N} TTT_i^T$ where TTT_i^T is the total system travel time during time period T on day i and N is the number of days.

Because of the system characteristics of the total travel time measurements, the mean of the total system travel time can be estimated from the statistics obtained on individual links in the system as follows:

$$TTT_s^T = \sum_{=1}^{L} TTT_\ell^T$$

where TTT_s^T, as previously defined, is the mean total travel time in the system in time period T in a sample of several days and TTT^T is the mean total travel time on link ℓ during the time period T on the same days. This states mathematically that, during time period T, the mean total travel time in a system equals the sum of the mean travel times on all of the L links in the system.

For each link ℓ during time period T there will be a natural variability in the total travel time from day to day. Thus, total travel time, if sampled on each of n_ℓ days, the variance of TTT_ℓ^T will be

$$\text{var } TTT_\ell^T = \frac{1}{n_\ell - 1} \sum_{i=1}^{n_\ell} \left[TTT_{\ell,i}^T - TTT_\ell^T \right]^2$$

$$= \frac{1}{n_\ell - 1} \left\{ \sum_{i=1}^{n_\ell} \left(TTT_{\ell,i}^T \right)^2 - n_\ell \left(TTT_\ell^T \right)^2 \right\}$$

In this equation, $TTT_{\ell,i}^T$, of course, represents, the total travel time on link ℓ during the time period T on day i. From the series of samples $TTT_{\ell,i}^T$ it would be possible to determine TTT^T and var TTT_ℓ^T. These could be determined for each link ℓ in the system.

From these measurements, the variance of total travel time in the entire system during time period T can be determined from the following relationship:

$$\text{var } TTT_s^T = \sum_{\ell=1}^{L} \text{var } TTT_\ell^T$$

SAMPLING PROCEDURES

Aerial Photographic Sampling Procedure

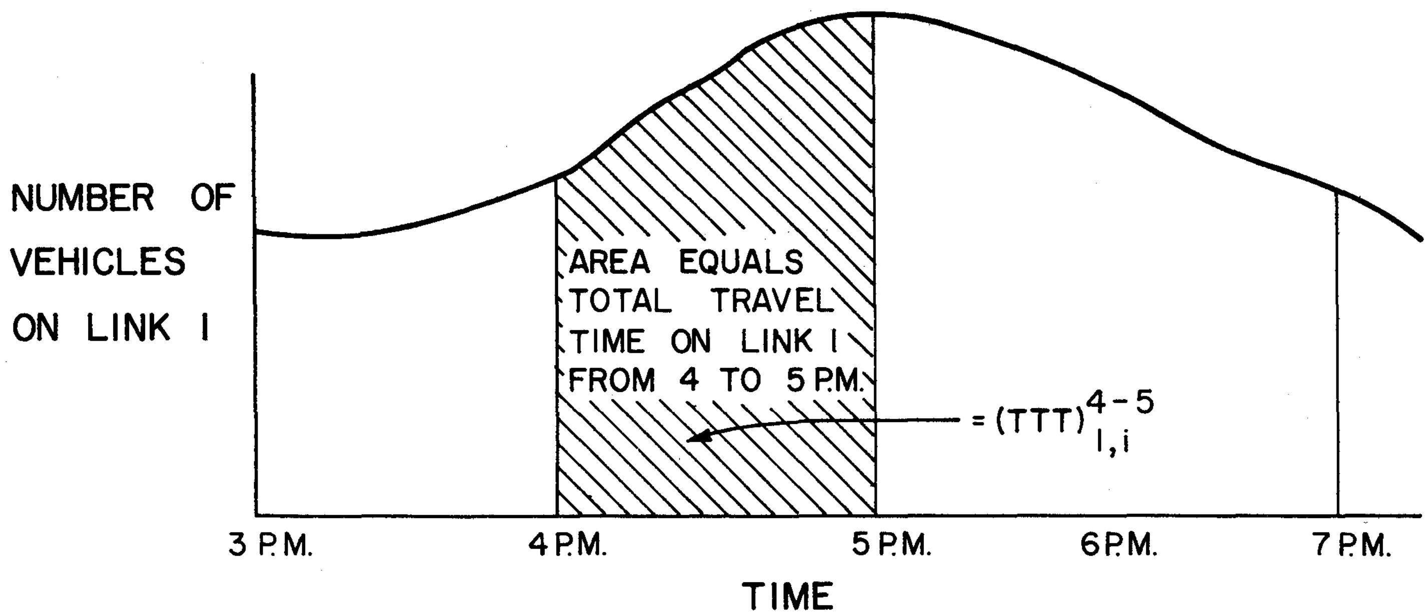
a. Sampling on Links

On any link the total travel time can be determined if the number of vehicles on the link as a function of time is known. Figure 4 illustrates, in theory, the procedure. In any time period, the total travel time on a link is the area under the curve of number of vehicles on the link plotted against time (1, 2).

In a network, this function of number of vehicles on a link is not known as a continuous function of time and must be estimated for each link. Samples must be taken in order to estimate the shape of this curve and it is necessary to determine the required frequency.

These samples can be obtained in several ways but, in many cases, they are most easily obtained by aerial strip photography. Thus, one sample would be obtained each time the airplane flies over the link. Figure 5 shows graphically this sampling procedure. Each dot on the curve represents a sample of the number of vehicles on the link obtained from aerial photography. Figure 6 shows a sample air photo of an arterial street in Detroit on which this type of analysis has been performed.

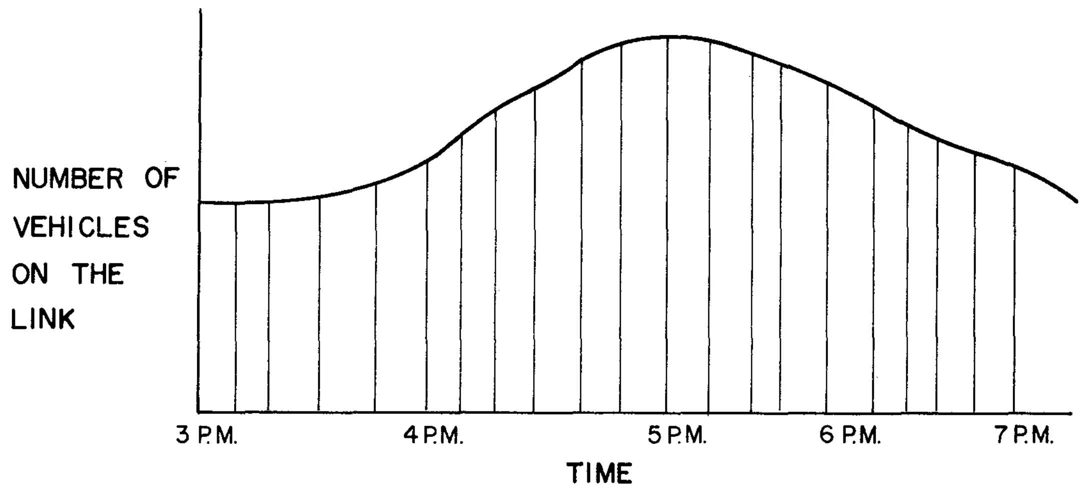
An increase in the frequency of flights over any link increases the accuracy of the estimate of travel time on the link on that day, but it also decreases the number of links that can be covered on any particular day. For a study of fixed or limited duration, this would, in turn,



METHOD OF DETERMINING TOTAL TRAVEL TIME
ON LINK L DURING ANY TIME PERIOD

FIGURE 4

decrease the number of days of observation of total travel time and this point will be discussed more completely in the next section.



SAMPLING PROCEDURE FOR ESTIMATING THE CURVE OF NUMBER OF VEHICLES ON A LINK AS A FUNCTION OF TIME

FIGURE 5

b. Sampling in a System

For each link ℓ in the system, a set of observations of total travel time during time period T will be known on day i and are denoted $(TTT)^{T}_{\ell, i}$. On a given day, if the frequency of flights or number of samples in the system is decreased, the number of links covered can be increased. Thus, the number of $TTT^{T}_{\ell, i}$ will be increased but each will be known with less accuracy.

The data presently being collected by the Texas Transportation Institute in Detroit will be quite useful in estimation of variances in total link travel time (var $TTT^{T}_{\ell, i}$) and can be used to design better sampling procedures in the future. Subjective estimates of these

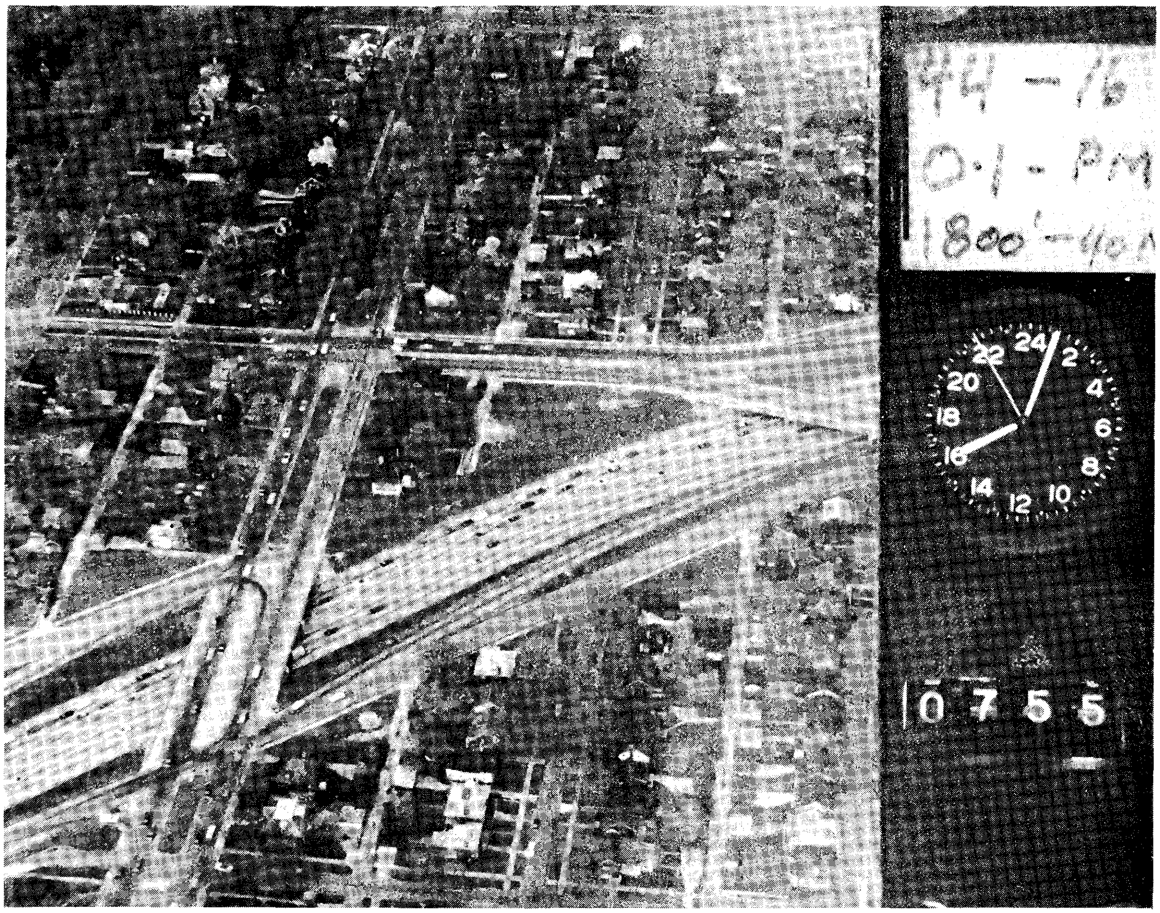


Figure 6. Example of Aerial Photograph Used To Evaluate Network Operation.

variances were used to design a set of airplane flight plans and sampling procedures for the initial research work and this is essentially a stratified sampling procedure based on the subjective estimates of var $TTT^T_{\ell,i}$.

MOVING VEHICLE SAMPLING PROCEDURES

A vehicle or several vehicles moving through a network can provide a great deal of information on the stream flow characteristics. By a sufficient sampling procedure and by combining the moving vehicle data for each link with the link volume data it is possible to perform a network analysis.

Let us examine the procedure for estimation of total travel time in the system. This estimation would require two types of data the link travel times and the link volumes. If $T^T_{\ell,i}$ is the travel time on link ℓ on day i in time period T the mean link travel time during period T is $T^T_\ell = \sum_{i=1}^{n_\ell} T^T_{\ell,i}$. The mean volume on link ℓ during time period T is $Q^T_\ell = \sum_{i=1}^{n_\ell} q^T_{\ell,i}$. The mean total travel time for link ℓ during time period T can be estimated as $TTT^T_\ell = Q^T_\ell \, T^T_\ell$.

The total travel time in the system can be estimated by $TTT^T_s = \sum_{\ell=1}^{L} Q^T_\ell \, T^T_\ell$.

Other traffic system measures which can be expanded from single-vehicle data are also obtainable from this method. For example an estimate of total acceleration noise expended in a traffic system can be estimated in this way.

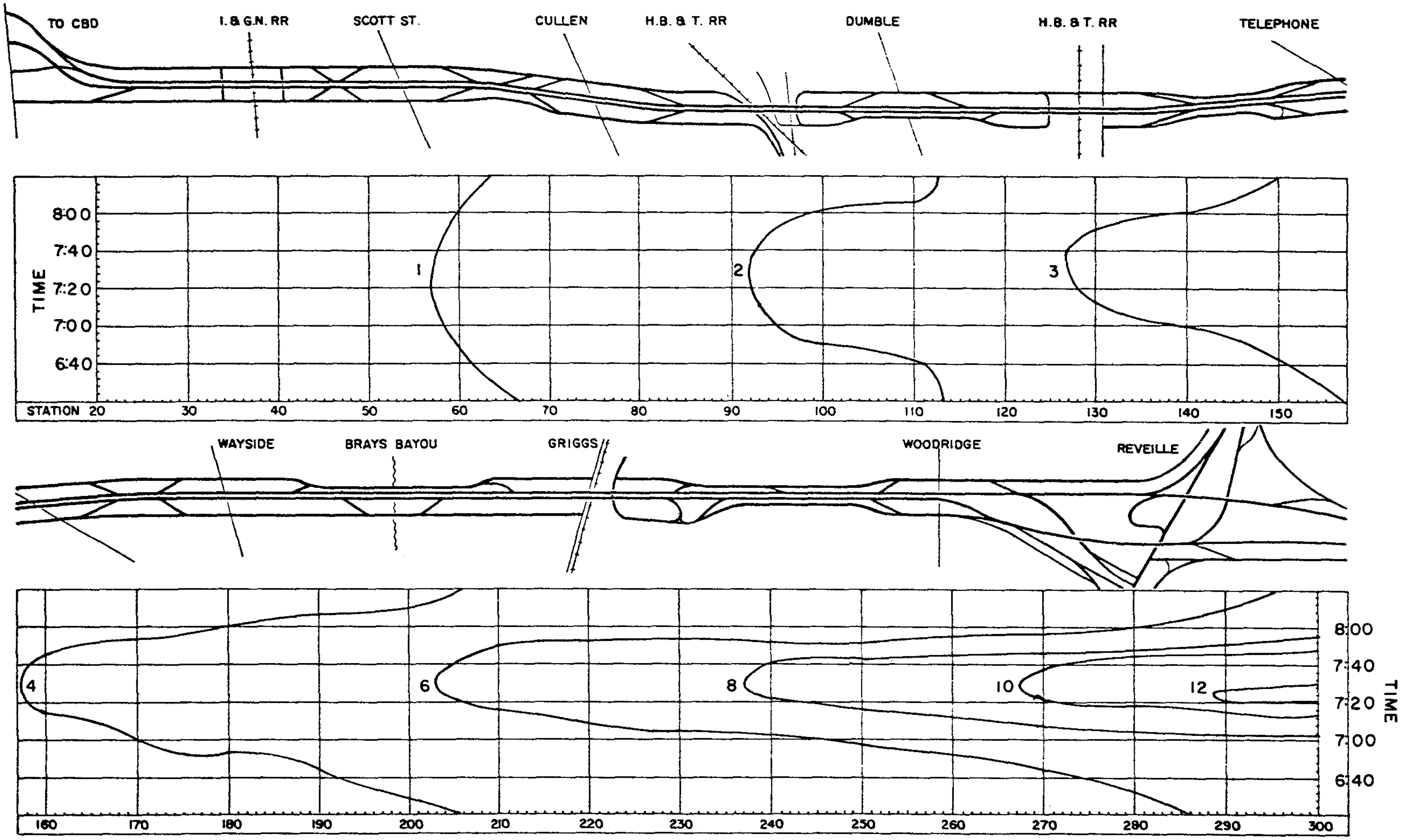
The moving vehicle method offers several advantages over the aerial photographic technique. First, the moving vehicle method can be used in any environmental condition while good weather is required for aerial photography. Secondly the cost of operation of a moving vehicle is less than that of an airplane. The airplane, because of its higher speed, leads to the collection of much more data per unit time by aerial photography.

Because the system figures of merit which are obtained by the moving vehicle method are the products of two random variables (such as link volume and link travel time) and because of the covariance between these variables, no relationships for the parameter variances have been developed.

SINGLE ROUTE ANALYSIS

The foregoing discussions have dealt primarily with traffic system operation. There are frequently instances, however, when one is interested in the behavior of traffic on a particular route or perhaps a series of parallel routes. This analysis may supplement the system analysis.

Classically, the traffic measures for one route have been presented in the form of contour maps such as the one in Figure 7. Most often the contour lines are not analytically fit to the data, but are merely estimates. This leaves little possibility for adequate statistical



AVERAGE TRAVEL TIME (MINUTES) TO END OF FREEWAY
CONTOUR MAP
DURING CONTROL — AUGUST 6, 7, 12, 13, 1964

Figure 7

devised using cells or rectangular fraction of the hypothetical surface such as the cell defined by its midpoint [P (4.5, 5:00)] in Figure 8. These cells are defined by time "slices" of 15 or 30 minutes and distance "slices" of 1/10 mile. If the time period of interest is from 2:30 to 6:30 PM (30 minute "slices") and the freeway study section is 8.6 miles long, there are a total of 688 cells for a given condition. These then can be tested against one another for significance.

A graphic representation of a hypothetical cell is shown in Figure 9.

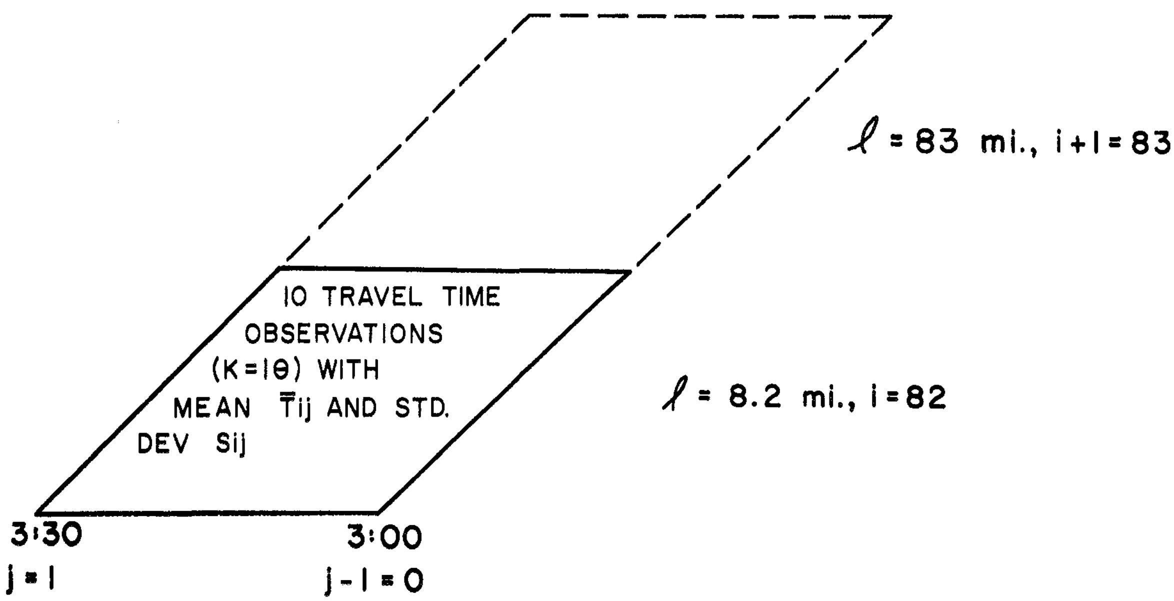


Figure 9. Cell Concept, Travel Time

When the averages and standard deviations have been computed for all cells and for all conditions they may be tested, assuming normality, cell by cell by the hypothesis, H: $\overline{T}^{n}_{ij} = \overline{T}^{c}_{ij}$. The form of the test is

1) $$t = (\overline{T}^{n}_{ij} - \overline{T}^{c}_{ij}) \,/\, S_{\overline{T}^{n} - \overline{T}^{c}},$$

Where $\overline{T}_{ij}^{n}$ = mean of cell ij for condition n,

$\overline{T}_{ij}^{c}$ = mean of cell ij for condition c,

$S_{\overline{T}n - \overline{T}c} = \left(\frac{S^2}{N_n} - \frac{S^2}{N_c} \right)^{\frac{1}{2}}$ and

$$S^2 = \left[\sum_{k=1}^{N_n} \left(T_{ijk}^{n} - \overline{T}_{ij}n\right)^2 + \sum_{k=1}^{N_c} \left(T_{ijk}^{c} - \overline{T}_{ij}c\right)^2 \right] \Big/ (N_n + N_c - 2)$$

The hypothesis is rejected if

2) $t \gtreqless \pm t_{(1-\alpha/2)(n_n + n_c - 2)}$

This assumes, also, that $S_n = S_c$. If this is not true another similar test is applicable. The "paired data" test may be applicable as well.

These statistics may be computed for travel time, speed and acceleration noise.

Two means of presentation may be used. The first are tables showing a two dimensional array of information as shown in Table 2 (for one cell).

TABLE 2

SAMPLE PRESENTATION FOR ONE CELL

	TIME, j
	k = n, sample size
Distance, i	$\overline{T}_{ij}$, S_{ij}

This will be done for any control condition presented. Secondly an array of test results showing significant differences between cells may be presented, Table 3.

TABLE 3

SAMPLE RESULTS OF TESTS

TIME

	NO	NO	NO
	NO	YES	NO
DISTANCE	YES	YES	NO
	YES	YES	YES

In this manner the differences in operating characteristics may be shown graphically. The critical points of variation between operating conditions (such as two methods of control) are quickly detected.

NETWORK INFORMATION SYSTEM

In order to organize the data which is required for a complete network analysis it is necessary to develop a network information system to facilitate the data handling and processing. In this way data from the various sources (air photos, ground counts, moving vehicle, etc.) can be integrated in a meaningful way.

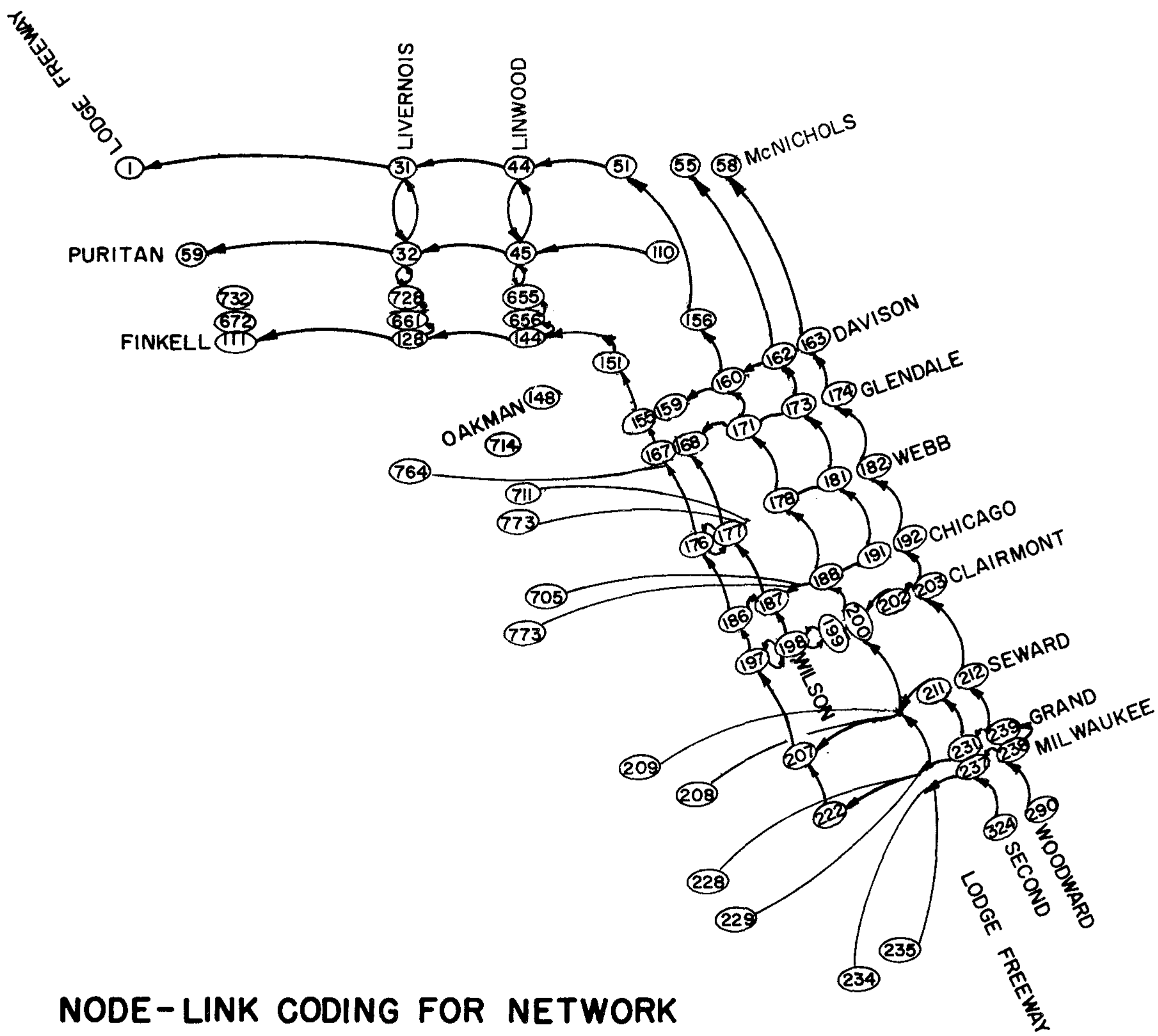
In order to facilitate the data handling on NCHRP Project 20-3, the Texas Transportation Institute developed a network information system. Figure 1 shows the traffic network in the vicinity of the John Lodge Freeway which was included in the analysis. All of the nodes in this network were assigned a unique code number so that each of the nodes and directional links are uniquely defined. Figure 10 shows the coded network and the directional links which are included in the system.

With this system excellent organization was possible in the collection of the data of which there was a great deal. This type of network coding also makes the network more easily amenable to more conventional analyses such as traffic assignment.

SUMMARY

The analysis techniques presented in this paper are extremely useful for operational analyses. The techniques are useful and are being used and at least temporarily fulfill the need for analysis procedures until more advanced and sophisticated techniques are developed. More advanced and more sophisticated techniques of network analysis are sorely needed.

The analysis of operational changes in large networks is an area in which a great deal of additional research is required. It is also a very important area because the operational evaluation of the effects of new control systems, new operational procedures, etc., are sadly lacking at the present time. Consequently, traffic engineers are forced to make decisions on new signal system purchases, new street operation plans and many other major programs without the benefit of prior evaluations of similar programs.



NODE-LINK CODING FOR NETWORK INFORMATION SYSTEM

Figure 10

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