## FRACTURE IN STABILIZED SOILS




## RESEARCH OBJECTIVES

## GENERAL OBJECTIVES

Based on the experience gained during the first one and one-half years of research, the revised research objectives of this study are described below, task by task.

## Task A - Fracture Analysis

1. Investigate laboratory procedures which can be accurately and sensitively used to verify and investigate the cracking initiation and propagation phenomenon occurring in fine grained media stabilized with portland cement. Develop, if necessary, new specimen geometries to produce reliable results.
2. Predict and verify the mode of crack initiation and propagation in soil cement. Classical linear elastic and elastic-plastic fracture mechanics theories of crack initiation and propagation will be investigated initially. Laboratory testing (Task A1) will verify whether or not these theories adequately explain the phenomenon. Theories accounting for the effects of microcracking and crack coalescence will be investigated by the process of developing a hypothesis followed by experimental verification of the hypothesis.
3. Determine the effects of matrix suction and osmotic suction as well as temperature on the fracture of stabilized fine grained media. The hypothesized effects of these variables are being verified by the tensile creep test. It is hypothesized that the effects of temperature, matrix and osmotic suction can be translated into shift
factors from the creep compliance curves similar to the classic time-temperature shift factor for polymeric material.
4. Establish the mode of fatigue crack growth in cement stabilized fine grained media. The effects of variable amplitude stress intensity fluctuations such as those caused by random loading spectra, externally applied thermal energy, kinetic energy of material in crack extension and other environmentally induced energy transfers will be investigated.

Task B - Analytical Fracture Study

1. Accurately model the laboratory test specimens selected for fracture testing in order to analytically predict the fracture properties of the materials tested.
(Note: Laboratory test experience during the first year of study proved the initially considered laboratory specimens to be unacceptable. Specimens traditionally used in fracture mechanics testing of metals were adopted instead. Thus, the work in Task B has been reduced to the verification of analytical predictions of stress intensity factors based on numerical predictions by means of finite element models.)
2. Accurately model stabilized soils when used as layers in a pavement structure and use material fracture properties to predict the number of cycles of load or environmentally induced stresses necessary to initiate and propagate cracks in the layer.
3. If the stabilized materials studied exhibit bimodular behavior, modify the fracture and structural analysis program to
accept bimodular material characterization.
4. Develop a simplified model based on fracture mechanics data and finite element analysis that can serve as a design/analysis tool to examine crack propagation in pavement structures.
5. Based on the fracture failure criteria established from laboratory testing of stabilized soils and using the analytical structural model discussed above, develop an algorithm to determine aircraft (load) equivalency factors for soil stabilized layers.
(Note: Tasks B2 through B5 are supplementary to the primary research effort. These tasks are supportive in nature. The University of Illinois has been very successful as a subcontractor in helping to accomplish these objectives through the development of finite element computer models during the first two years of this contract. However, in order to keep within the guidelines of basic research, we do not intend to extend the Task B study into the third year of the contract. We are confident that Task B will be successfully accomplished by April, 1984.)

Task C - Study Effects of Soil Fabric, Mineralogy and Chemistry on Fracture Properties

## STATUS OF RESEARCH EFFORT

PROGRESS OF FRACTURE ANALYSIS (TASK A)
The development of the acceptable energy methods for the measurement and verification of crack growth in a cemented fine grained media is of critical importance to this research effort. The fracture mechanics techniques being used have never been proven to be acceptable procedures for this type material. Fortunately, the validation of these energy methods for cement stabilized fine grained media provides perhaps the best possible insight into the mode and process of fracture occurring within the media.

The following paragraphs discuss the progress made in the development of energy methods which can be used to verify and quantify the fracture process. The researchers are confident that, based on the progress to date, the objectives of Tasks $A, B$ and $C$ can be accomplished in the three year period originally proposed for this contract.

Qualitative Discussion of Energy Methods (Tasks A1 and A2)
Most elementary texts (e.g. Broek (1983)) on fracture contain discussions of several approaches for calculating the energy required to produce crack extension in fracture and for calculating critical stress intensity factors. The two methods used in this report are the Griffith (1920) and Irwin (1948) method the Rice (1968) and ASTM (1981) method.

Applicability
In the linear elastic case, both the Griffith criteria (G) and the Rice approach (J integral) are identical. Both methods yield a numerical value which describes the energy release per unit area of crack extension. If properly used, the $J$ integral method can also be used in the non-linear elastic case and the elastic-plastic case.

Cement stabilized fine grained soils behave in a linear elastic manner. All of the specimens discussed in this report contain $10 \%$ portland cement. Higher cement contents may not behave quite as nearly linear elastic. The $10 \%$ specimens showed a very brittle fracture surface under the microscope. Tensile bars pulled to failure (unflawed specimens) exhibited nearly perfectly elastic stress-strain behavior.

The use of linear elastic fracture mechanics (LEFM) is limited to the case where the value of crack tip radius, $\rho$, is less than that radius at which the cohesive strength (force-equilibrium) ( $\sigma_{f c}$ ) vs crack tip radius ( $\rho$ ) curve intersects the available energy/thermodynamic limited strength ( $\sigma_{f E}$ ) vs $\rho$ curve (see Figure 1). Typically, this requirement is assumed to be met if precracking is successful, or if the radius of curvature of the crack tip can be estimated based on interatomic spacing. In this case, it is reasonable to modify this latter approach somewhat. It was considered that the starter notch had a $\rho$ on the order of the size of the soil particles. Since almost 100 percent of the soil matrix was finer than the No. 100 sieve, the nominal radius of curvature was computed to be
approximately, $150 \times 10^{-6} \mathrm{~m}$ or $5.9055 \times 10^{-9} \mathrm{in}$. Failure of the samples (as supported by microscopic visual inspection) occurs through the binder/void regions between soil particles. Therefore, the intermolecular spacing, $a_{0}$, of the linked-tetrahedral cement structure is assumed to be the critical spacing. The value of $a_{o}$ was assumed to be 2.5 angstroms based on values of interatomic spacing of elements expected within and between the tetrahedral sheets.

The cohesive strength, ${ }_{c}$, was estimated using the relationship $\sigma_{C}=E / \pi$ (where $E$ is the modulus of elasticity, psi). This relationship gives a $\sigma_{C}$ which is slightly higher than that exhibited in real materials. However, this analysis is meant only to be an order of magnitude analysis, and the approximations required are considered acceptable. Therefore, $\sigma_{C}=583,000 / \pi=185,575$ psi (E was determined from direct tensile testing). The energy per unit area, $\gamma$, is ordinarily assumed to have a plastic component, $\gamma_{p}$, and an elastic component, $\gamma_{s}$. Because of the near perfect elastic and brittle behavior of this material, $\gamma_{p} \doteq 0$. Therefore,

$$
\gamma_{S}=\frac{2 E a_{0}}{\pi^{2}} \doteq 2.95 \times 10^{-5} 1 \mathrm{~b} / \mathrm{in}
$$

(Derived from Bradley (1983))
and for cohesive strength

$$
\sigma_{f c} \sqrt{a}=\left(\frac{E_{\gamma} s^{p}}{4 a_{0}}\right)^{1 / 2} \doteq 10 \text { psi } \sqrt{i n}
$$

and for energy limited strength

$$
\sigma_{f E} \sqrt{a}=\left(\frac{G E}{\pi\left(1-v^{2}\right)}\right)^{1 / 2}=131 \mathrm{psi} \sqrt{\mathrm{in}}
$$

Where $a=$ crack length and $\nu=$ Poisson's ratio.
Since $\sigma_{f c} \ll \sigma_{f E}$, LEFM is applicable.
Deviations From Theory
In general, no significant deviations from LEFM theory are apparent in the specimens. Heterogeneity of the samples is a topic which is discussed in the $J$ integral section of the report.

Energy Determinations (Tasks A2 and A2)
J Integral Analyses
In general, J integral analyses showed more variation between individual specimens but less variation overall than the $G$ determinations. More study in this area is required because of the fact that the $J$ integral procedure should theoretically give better energy determinations at the higher cement contents which will be studied in the future (i.e. the J integral procedure will be necessary if non-linear behavior is observed at higher cement contents).

Blunting Line Discussion
The blunting line is a line which describes the $J$ integral vs change in crack length behavior for a "pseudo" crack extension attributed to an attempt by the material to maintain constant volume at the crack tip immediately before actual material separation at the
crack tip. For materials with high fracture energies and high flow stresses, the blunting line often gives ${ }^{J}$ IC values which are significantly higher than values that would be obtained with no consideration of a blunting line. For the soil cement materials. studied, very little blunting takes place. For all materials, the choice of the blunting line equation in ASTM E813 is often critical and not necessarily constant. Dugdale (1960) treated the plastic zone as a strip, assumed plane stress and a through crack in a wide plate. The validity of these assumptions is questionable when using the ASTM E813 procedure. Dugdale showed that

$$
\delta=\frac{8 \sigma y^{a}}{\pi E}\left[\frac{1}{2}\left(\frac{\pi \sigma}{2 \sigma_{y s}}\right)^{2}+\frac{-1}{12}\left(\frac{\pi \sigma}{2 \sigma_{y s}}\right)^{4}+\frac{1}{45}\left(\frac{\pi \sigma}{2 \sigma_{y s}}\right)^{6}+\ldots H 0 T \ldots\right]
$$

Where $\delta$ is the crack tip opening displacement (CTOD) and $\sigma_{y s}$ is the yield strength. For $\sigma / \sigma_{y s}<3 / 4$ the high order terms are small. This assumption is acceptable for the material studied and from Burdexin. and Stone (1966):

$$
\delta=\frac{\pi \sigma^{2} a}{E \sigma_{y s}}
$$

FOR $\quad K_{1}=\sigma \sqrt{\pi a} \quad \Rightarrow \quad K_{1}^{2}=\sigma^{2} \pi a$

$$
\Rightarrow \quad \delta E \sigma_{y s}=\pi \sigma^{2} a
$$

$$
k_{1}^{2}=\delta E \sigma_{y s}
$$

SINCE $E=\sigma_{y s} / \varepsilon_{y s} \quad$ THEN $\delta / \varepsilon_{y s}=\left(K_{1} / \sigma_{y s}\right)^{2}$
BUT $K_{1}^{2}=G E \Rightarrow G=\delta \sigma_{y s}$
AND FOR LEFM $J=G$

$$
\Rightarrow J=\delta \sigma_{y s} \quad \text { (PLANE STRESS) }
$$

To correct to the plane strain case, we must introduce a factor, $M$, such that $J=M \delta \sigma_{y s}$. ASTM E813 specifies the blunting line as follows: crack advance $=\delta / 2=\Delta a$ and for $M=1, J=\delta \sigma_{y s}$, $\delta=2 \Delta a$, $\Rightarrow J=2 \Delta a \sigma_{y s}$ which is the form of ASTM E813. In reality we have plane strain $\Rightarrow M \neq 1$
$\therefore J=2 M \Delta a \sigma_{y s}$
$\sigma_{y s}$ is the flow stress which is 160 psi for the cement stabilized fine grained soil ( $\sigma_{y s}=\sigma_{U T S}$ (Ultimate Tensile Strength) for this brittle material).

The determination of $M$ for high toughness/ductile/nonlinear materials is more critical than for the cement stabilized material where little blunting is expected. The effect of material behavior on the blunting line will be studied more during the next few months (Spring of 1984).

Use Of ASTM E813 Specification
The use of the standard test procedure blunting line in conjunction with various curve fits to the data yielded values of ${ }^{3}$ IC ranging from 0.077 to $0.101 \mathrm{lb} / \mathrm{in}$ (see Table 1 and Figures 2, 4, 6, and 8).

Use Of Data As Blunting Line
In the event of "psuedo" crack extension due to crack tip blunting, one might expect accurate crack length measurements to identify this trend. Thus, data could be used to determine the blunting line. Using the data to define the blunting line yielded values of crack tip blunting of up to 0.1 inch. However, considering the basic physical concepts involved, it is obvious that this is not the proper procedure because blunting of the crack tip by up to 0.1 inch in this material is unreasonable.

No Blunting Line Analysis
If it is assumed that no blunting occurs at the crack tip, the intercept of a simple regression curve fit to the data could be used as ${ }_{\text {IC }}$. This method seems reasonable for this low fracture energy material. This method yielded JIC values essentially the same as found when a blunting line was used.

## R-Curve Fitting Procedures

The crack resistance (R-Curve) concept is discussed in Broek (1983). In the $J$ integral procedure, the $J$ vs $\Delta$ data essentially define an R-Curve. For plane strain conditions the slope of the line through the data is approximately zero (note that our $R$-curves have slopes close to zero, i.e., . 09 to . 15). For plane stress, a parabolic shaped curve results and is above the plane strain R-Curve on the $J$ vs $\Delta a$ plot. Intermediate stress states result in corresponding changes in the R-Curve. From this point forward, the
terms R-Curve and J-Curve will be used interchangeably.

Linear Fit
In using a linear fit to the $J$ vs $\Delta a \operatorname{data}$, it was noted that the data were not really linear but had a stair-step appearance, with some portions of the curve having shallower slopes than other portions. The physical explanation for this behavior is associated with material heterogeneity. The obvious conclusion for the apparent discontinuity of the $J$ vs $\Delta$ a cupve is that portions of the soil-cement material experience stronger bonding than other portions. Another possible explanation that is not quite so obvious begins with the assumption that all stored elastic energy is released during crack extension. In actuality, the energy release during crack extension may not be completed with each extension. The difference between the energy input into the system and the energy released per unit area crack extension represents a build-up of "reserve" stored elastic energy. With a critical combination of reserve energy and new, externally applied energy put into the system, two possible reactions may occur. First, a sudden crack extension occurs in which all stored energy is released (i.e. a larger a occurs than expected at a nearly constant $J$ value). Then the energy storage process begins again. Alternatively, the reserve energy may be gradually released with further externally applied energy/ $\Delta$ a combinations until the reserve is depleted to zero (or at least a low level). This explains the temporary decrease in the slope of the $J$ vs $\Delta$ a curve because a lower external energy application is required to achieve the same crack extension, $\Delta \mathrm{a}$. When
the reserve energy storage process begins again, the slope of the $J$ vs $\Delta$ curve increases to the original slope (or slightly less than the original due to finite size effects on the stress state and/or due to the inability of the storage reservoir to fully reach the zero energy level).

The material heterogeneity explanation can be conveniently supported with existing theory. The reserve energy storage idea can not be supported with our present level of understanding.

For limited ranges of $\Delta a$, the stair-step nature of the $J$ vs $\Delta a$ curve may cause $J_{I C}$ to vary significantly. Over a wide range of $\Delta a$, the variation in $J_{\text {IC }}$ due to the change in the intercept and slope of the fitting equation may be acceptable.

Parabolic Fit
A parabolic form was also used to fit the data (see Figure 10). ${ }^{J}$ IC was taken as the intercept. Good agreement was found between J IC determined from the parabolic form, and that determined using the ASTM E813 blunting line and using only the data points which described the initial steep slope of the J-Curve. The agreement was only good within a single specimen. The internal agreement would be expected due to the mathematical behavior of the parabolic function. However, the values were much lower than the $G$ values and were, therefore, not used.

G Analyses
The original Griffith (1970) equation for critical energy release rate can be taken as

$$
G=\frac{p^{2}}{2 B} \quad \frac{\partial C}{\partial a}
$$

Where $P=$ critical load, $B=$ sample thickness, and $C=$ compliance. The identification of the critical load for crack extension is not a problem when using a "Krak" gage for crack length measurement.

Method of Determining Compliance
The method of compliance determination is difficult and critical for this method of energy analysis. Normally compliance measurements are taken by measuring $d \delta / d P$ in the linear portion of the unload-reload P- $\delta$ plots (Figure 11). It is assumed that if the unload recovers to the origin $((\delta, P)=(0,0))$ along a straight or curved line, the material is elastic. If, at $P=0, \delta>0$; the material has experienced permanent deformation without crack extension (i.e. it is plastic) and $G$ does not apply. It should be noted that ASTM E813 recommends an unload of only $10 \%$ which corresponds to 2 to 4 pounds for this material. Unloads of greater than this amount may adversely affect the analysis. A hysteresis type loop (possibly due to microcracking or crack closure mechanisms) appears in the cement stabilized fine grained soil in this case. Determination of where on the loop to make the compliance measurement then becomes a problem.

Method D
In method $D$, a small amount of plastic deformation and/or non-linear behavior was assumed to have occurred. A relatively linear portion of the reload compliance curve was used to compute the compliance at each point. Theoretically $G$ does not apply in this case. Although there was some agreement in $G_{\text {IC }}$ values found using this method, the values were much lower than any of the $J_{\text {IC }}$ values. Since the material behaves in a linear, elastic, brittle manner; this method of analysis is not considered acceptable.

## Method A

Assuming linear elastic behavior, a straight line was constructed from the origin to any ( $\delta, P$ ) coordinate pair and compliance was calculated directly,

$$
C=\delta / P .
$$

Method A (see Figures 3, 5, 7, and 9) produced excellent results. In addition, this procedure may be applicable to other materials which exhibit hysteresis loops. Magnification of that portion of the $P$ - $\delta$ curve between the start of the unload and the completion of the reload reveals a rather non-linear plot (Figure 11a). In this region the ( $\delta$, P) coordinate pairs for the unload and for the reload are equal at only two locations (i.e. the equations describing the unioad and the reload could be simultaneously solved at only two points). These points are the point of load reversal and the point (at a load greater than reversal) where the unload and reload plots intersect. The load reversal point has a $(\delta, P)$ coordinate pair which is not unique (i.e.
it is a function of how far the unload is allowed to progress). On the other hand, the intersection of the unload and reload curves is a unique point in the single hysteresis loop. Now, by superposition of a continuous P- $\delta$ plot (with no unloading for compliance measurement) over the magnified curve (Figure 11b), a range of ( $\delta, P$ ) data pairs is available for determination of compliance. The proper coordinate pair for the point at which compliance is to be measured is selected as follows:
(1) Determine the load at which the unload starts.
(2) Determine the peak load which occurs on the reload.
(3) Take the average of the two loads.
(4) Find $\delta$ at the average load determined above as defined by the continuous $\mathrm{P}-\delta \mathrm{plot}$.

After the point on the continuous $\mathrm{P}-\delta$ plot and the unload-reload intersection have been defined, draw a straight line through these two points and back to the coordinate system axes. For a linear, elastic, brittle material, the line will intersect the origin. The inverse slope of this line is related to the compliance. The calculation of $\partial C / \partial a$ is derived from differentiation of a curve fit to the $C$ vs a curve. Several facts are known concerning the fitting equation. At high a values, the equation has an exponential form. At $a=0$, it would seem reasonable to expect that the value of $C$ would be $1 / E$ and the slope $\partial C / \partial a$ should be zero at that point. Accordingly, an equation of the form

$$
\ln C=\ln (1 / E)+a(\ln A)
$$

was used. The intercept was forced to be close to $1 / E$ and the slope was found to be sufficiently small at that point to be adequate for our purposes.

Geometric Considerations
Measurements for displacement on the compact tension specimens were taken at the front face of the specimen. Displacement should actually be measured at the load line. The correction for the measurement location was made using the techniques summarized by Saxena and Hudak (1977).

Comparison of $G$ and $J$ Determinations
Very consistent, reproducible results were obtained using the $G$ determination procedure. The J integral technique yielded comparable energy values.

Possibilities for Refinement in Analysis Technique
Further research is necessary to refine the $J$ integral procedure for this material. Specifically, refinement of the procedure for determining which data points should be used to determine the R-Curve is necessary. It is felt that the multiple unloads for compliance measurements may be causing problems in the analysis which increase the J IC variability due to effects on the slope and intercept of the fitting line. In order to solve these problems and to determine if this is what is happening, one sample per point as in the original J IC method will be used to determine the $J$ vs $\Delta$ a curve. This will require precracking. We have successfully precracked C-T specimens. The most
significant problem with the precracking procedure is visually determining the position of the crack front. Several solutions to this problem are available (Krak gage, surface coating, or compliance).

A very minor source of error may be the geometric correction for our measurement location. In order to assess the severity of this possible source of error, several three point bend specimens will be tested. Measurements are taken at the load line for the three point bend specimen. Therefore, no correction is required. Another minor source may be the use of the trapezoidal rule for integration of the area under the curve and not recalculating $f(a / W)$ for each a. This error is considered insignificant.

## Fracture Toughness Inferences

Visual inspection of the specimens confirm that the plane strain stress state exists essentially through the sample. Therefore, the calculation of $K_{I C}$ from $G_{I C}$ was taken as the plane strain value with $v$ assumed to be 0.15 :

$$
K_{I C}=\left(\frac{G_{I C^{E}}}{\left(1-\nu^{2}\right)}\right)^{\frac{1}{2}}
$$

$K_{\text {IC }}$ was found to vary between 213.7 and 245.8 psi $\sqrt{i n}$.

Energy Methods - Summary and Conclusions (Tasks A1 and A2)
The research in Task $A$ has proven that the compact tensile specimens with chevrons typically used for fracture testing of metals can produce reliable and reproduceable results for cement stabilized
fine grained soils. Several quite lengthy and time consuming experiments were necessary in order to prove that the results of the fracture testing were indeed valid and did not violate the fundamental prịnciples of fracture mechanics. One particularly noteworthy accomplishment was the ability to establish the range of the critical stress intensity factor for the portland cement stabilized fine grained soil studied. The critical stress intensity factor was found to vary between 213.7 and 245.8 psi $\sqrt{\mathrm{in}}$ for a specific fine grained soil stabilized with $10 \%$ portland cement. This experimental procedure can now be used to precisely differentiate among mixture variables and thus verify hypothetical effects on the mode and mechanism of fracture caused by select variables.

During the period of January through March, 1984, research will concentrate on the following topics:

1. Continue the study of the hysteresis loop developed during the unload portion of the compliance curve. The method now used for measuring the compliance is very satisfactory. However, the hysteresis loop probably has great significance in terms of fatigue and is thought to be related to microcracking and crack closure effects. This will be studied using microscopy together with compact tensile compliance measurements.
2. Begin the evaluation of fatigue related fracture using the compact tensile test as a verification tool to experimentally evaluate fracture hypotheses.

## Creep Testing (Task A3)

Creep testing has been conducted intermittently for the past one and one-half years, mostly with disappointing results. However, this testing is a critical link in the ultimate description of the monotonic and cyclic (fatigue) fracture phenomenon in cement stabilized fine grained soils. The ability to develop realistic shift factors for the compliance curves to explain the effects of osmotic and matrix suction, temperature and cement content is the key to experimental verification of the hypothetical influence of these factors.

It is believed that the creep effect is due more to a microcracking phenomenon and a crack coalescence phenomenon than to any viscoelastic effects in this unique material. Light and electron microscopy together with creep testing will be used to explain and verify the relationship between creep and fracture and the effects of the factors of cement content, temperature and suction.

Figure 12 presents the results of some recent creep testing.

## Fatigue Related Crack Growth (Task A4)

The annual report for this contract submitted on June 30, 1983, provides an in depth discussion of the classical methods to utilize fracture mechanics techniques to define fatigue induced fracture in homogeneous, elastic materials. Before these techniques can be used for cement stabilized fine grained soils, they must be verified. Most of the verification has been accomplished during this reporting period. However, several steps remain which will be accomplished in

## the spring.

Typically, recent research has concentrated on da/dN versus $\Delta K$ (the basic Paris relationship) as the basic approach to fatigue life estimation and as the indirect explanation of the fatigue fracture phenomenon. Thus failure and fatigue life have been directly related to the magnitude of $K$. Perhaps a better approach would be to relate failure to a critical energy level (e.g. J ${ }_{I C}$ or $G_{I C}$ ). Thus, instead of trying to relate a stress history to $d a / d N$ through the relationship of the stress history to a critical stress intensity, one could relate stress intensity to damage through a critical energy level concept.

It might be said that energy is proportional to the square of the stress intensity and, thus, there is no difference between the two methods mentioned above. However, the energy approach allows one to conveniently include other sources of energy not completely described by mechanically applied stress and specimen geometry. Some other components of energy of most concern when considering cement stabilized fine grained soils are:

1. Externally applied thermal energy $E_{T}$,
2. Kinetic energy of material in crack extension, $E_{K}$ and
3. Other energy transfers due to aggressive environments $E_{A}$.

As an example of the ease with which we might manipulate energy terms, consider the extension of the energy line integral (J) concept to the case of a summation of energy release during crack extension over time (for an appropriate volume). We begin with the equation for stress intensity at the end of the minor axis of a surface semi-
elliptical flaw. The equation includes the back free-surface correction and an approximation to an elliptical integral which describes the flaw shape parameter (Brook, 1983).

$$
\text { For } \begin{aligned}
K_{I} & =\frac{1.12 \sigma \sqrt{\pi a}}{\left(\frac{3 \pi}{8}+\frac{\pi a^{2}}{8 C^{2}}\right)} \\
& =\frac{15.88117 \sigma \sqrt{a}}{\left(3 \pi+\pi a^{2} / C^{2}\right)} \\
& =\frac{15.881187}{\pi} \frac{\sigma \sqrt{a}}{\left(3+a^{2} / C^{2}\right)} \\
& =\frac{5.0551387 \sigma \sqrt{a}}{\left(3+a^{2} / C^{2}\right)} \\
\Rightarrow K_{I}^{2} & =\frac{25.554427 \sigma^{2} \sqrt{a}}{\left(9+6 \frac{a^{2}}{C^{2}}+\frac{a^{4}}{C^{4}}\right)}
\end{aligned}
$$

And For $K^{2}=\frac{J E}{\left(1-\nu^{2}\right)}$

$$
\Rightarrow J=\frac{K^{2}\left(1-\nu^{2}\right)}{E}
$$

Where $a / c$ is minor to major axis ratio.
Therefore, with no additional sources of energy (egg. for a CT specimen in an isothermal vacuum): $s^{v} f^{t} J d t d v=s^{v} s^{t} \frac{K^{2}\left(1-v^{2}\right)}{E} d t d v$

$$
\begin{aligned}
& =\left(\frac{1-v^{2}}{E}\right) \rho_{\rho} t k^{2} d t d v \\
& =\frac{25.554427\left(1-v^{2}\right)}{E} \rho^{v} \rho^{t} \frac{\sigma^{2} a}{\left(9+6(a / c)^{2}+(a \neq c)^{4}\right)} d t d v
\end{aligned}
$$

Where $\sigma=$ stress, $v=$ volume, and $t=$ time. If $j^{v} f^{\ddagger} J d t d v>J_{\text {IC }}$ failure due to cracking results.

In the case of additional sources of energy, we might say failure occurs if:
$J_{I C}<\int^{v} \int^{t}\left(J \pm E_{T}+E_{K} \pm E_{A}\right) d t d v$
Intuitively, we suspect that

$$
E_{T}=f(T)
$$

$$
E_{K}=f(\dot{\varepsilon})=f(\dot{\sigma})=f(d \sigma / d t)
$$

$E_{A}=f$ (Bond strength change due to particle, molecular, and/or atomic scale chemical modifications).

Where $\mathrm{T}=$ temperature, and $\dot{\varepsilon}=$ strain rate.
The researchers believe that the analytical and experimental techniques being used and developed during this research effort will be able to effectively consider and verify these effects.

## PUBLICATIONS

The following technical publications are in preparation:

1. A paper which will discuss the determination of stress intensity factors based on the $J$ integral and the energy release rate approach using compact tensile specimens will be submitted to the journal, Experimental Mechanics. This paper is primarily a rebuttal and discussion of a paper published in the same Journal in November, 1982, entitled, "Stress Intensity Factor for Plain Concrete in Bending - Prenotched Versus Precracked Beams". The research for this paper was sponsored by a grant from the National Science Foundation. The rebuttal paper will be submitted for publication in February, 1984.
2. A second paper entitled "Measurement of Compliance in Brittle Materials Exhibiting Hysteresis Loops" will be submitted in early spring to Experimental Mechanics.

Following the establishment of fatigue fracture relationships which will be developed in the spring and in the third year of the contract, the following papers are planned and are being prepared in a preliminary manner:

1. "Extending Fracture Mechanics to Chemically Stabilized Fine Grained Soils Used as Structural Layers in Pavement Systems", D. Little, W. Crockford and W. Bradley, Transportation Journal of American Society of Civil Engineers.
2. "Computation of Aircraft Damage Factors Using Fracture Mechanics in Pavements Containing Cement Stabilized Fine Grained Soils", W. Crockford, D. Little and S. Carpenter, Transportation

Journal of American Society of Civil Engineers.
3. "A Comprehensive Fracture Mechanics Model to Predict the Service Life of Soil Cement Structural Layers", W. Crockford, D. Little and S. Carpenter, Transportation Research Record, Transportation Research Board.
4. "The Role of Microcracking in the Fracture of Cemented Fine Grained Soils", Y. Kim and D. Little, Journal Undecided.
5. "The Effects of Suction, Temperature and Binder Content on Tensile Load Induced Microcracking in Cement Stabilized Fine Grained Soils", Y. Kim and D. Little, Journal Undecided.

It is also hoped that Task $C$ of this contract will include the development of significant publications in the area of soil fabric and mineralogy and their effect on microcracking, creep and fracture of cement stabilized fine grained soils.

Table 1. Summary of Fracture Parameters.

| Method | $\begin{aligned} & \mathrm{a}_{0}(\mathrm{ID}) \\ & \mathrm{P}_{\max }(\mathrm{LB}) \end{aligned}$ | Prediction Equation | $R^{2}$ | $\begin{gathered} \mathrm{J}_{1 \mathrm{C}} \\ \mathrm{LB} / \mathrm{IN} \end{gathered}$ | $\begin{gathered} \mathrm{G}_{1 \mathrm{C}} \\ \mathrm{LB} / \mathrm{IN} \end{gathered}$ | $\begin{gathered} \mathrm{K}_{1 \mathrm{C}} \\ \text { PSI } \mathrm{ID} \end{gathered}$ | Sample \# |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{G}_{1 \mathrm{C}}$ | $\begin{gathered} 1.0413375 \\ 19.6 \end{gathered}$ | $C=\operatorname{EXP}(-13.19576019+4.20810596$ a) | . 995292 |  | . 0822373 | 221.47 | 2 |
| ${ }^{J} 1 \mathrm{C}$ |  | $\mathrm{J}=.10129222+.09249857 \Delta \mathrm{a}$ | . 918751 | . 1013214 |  | 245.83 | 2 |
| $\mathrm{G}_{1 \mathrm{C}}$ | $\begin{gathered} .962007 \\ 38.5 \end{gathered}$ | $C=\operatorname{EXP}(-13.5763334+3.8349574 \mathrm{a})$ | . 9817 |  | . 0965905 | 240.02 | 7 |
| ${ }^{J} 1 \mathrm{C}$ |  | $\mathrm{J}=.07655128+.15439217 \Delta \mathrm{a}$ | . 950853 | . 0765881 |  | 213.73 | 7 |
| $\mathrm{G}_{1 \mathrm{C}}$ | $\begin{aligned} & .972881 \\ & 29.6 \end{aligned}$ | $C=\operatorname{EXP}(-13.5018417+4.19024365 \mathrm{a})$ | . 982624 |  | . 0993105 | 243.37 | 8 |
| ${ }^{J} 1 \mathrm{C}$ |  | $\mathrm{J}=.09338438+.10619517 \Delta \mathrm{a}$ | . 973594 | . 0934153 |  | 236.04 | 8 |
| $\mathrm{G}_{1 C}$ |  | $C=\operatorname{EXP}(-13.63206476+3.89074601 \mathrm{a})$ | . 943906 |  | . 0960214 | 239.31 | ALL* |
| ${ }^{J} 1 \mathrm{C}$ |  | $\mathrm{J}=.08871491+.12804492 \Delta \mathrm{a}$ | . 915430 | . 0887504 |  | 230.07 | ALL |

[^0]

Figure 1. Graphical Representation of Crack Tip Radii For Which LEFM And/Or Stress Concentration Approaches Are Applicable (After Bradley (1983)).


Figure 2. J Integral For Sample \#2.


Figure 3. C vs a For Calculation of G, Sample \#2.


Figure 4. J Integral For Sample \#7.


COMPLIANCE VS CRACK LENGTH

Figure 5. C vs a For Calculation of $G$, Sample \#7.


Figure 6. J Integral For Sample \#8.


Figure 7. C vs a For Calculation of $G$, Sample \#8.


Figure 8. J Integral For All Data From All Specimens.


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Figure 9. C vs a For All Data From All Specimens.


Figure 10. Parabolic Fit To All Data, J vs a.


Figure 11. Load-Displacement Curves.
(a) Unloads Show Hysteresis.
(b) Compliance Determination For A Linear Elastic Material.


Figure 12a. Creep Curves Illustrating the Sensitivity to Cement Content.


Figure 12b. Creep Curves Illustrating the Sensitivity to Cement Content.

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[^0]:    *Average of three values $G_{I C}=.0516135, .146259967, .090190657$.

