

BACKCALCULATION OF PAVEMENT LAYER PROPERTIES

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INTRODUCTION

This paper briefly summarizes historical developments, reviews current conditions in the use of nondestructive testing (NDT) to determine the properties of pavement layers by backcalculation, and finally, looks forward to future developments and uses of NDT.

The properties of pavement layers that are needed for the evaluation, design, and management of pavements is more extensive than are presently being collected by the use of nondestructive testing. They include the following: a) layer thickness; b) binder content in asphalt bound layers; c) the elastic stiffness of each layer which means either the elastic modulus or the stress-strain curve properties of stress-dependent materials; d) fatigue properties for both load and thermal fatigue processes; e) permanent deformation properties of each layer; f) residual stresses in situ; and g) other properties. Accurate measurement of all of these properties is essential to making realistic predictions of the remaining pavement life and designs of overlays and recycled layers. Most of the properties in the list given above are not measured at present but there is no reason that means to measure them nondestructively cannot be found.

The most common property found by nondestructive testing is the elastic stiffness of each layer. Whether the elastic modulus or the ~~properties of the nonlinear stress-strain curve is chosen~~, primarily, it should be compatible with the method that is used to make design calculations: multi-layered elastic or finite element methods. To be

consistent, the same method should be used to make predictions of remaining life, for monitoring the change of layer properties with time, and for use in specification testing.

The nondestructive testing equipment used in making the measurements includes a variety of modes of applying loads to a pavement and a number of sensors for measuring the pavement response. The loading methods include: a) static or slowly moving loads; b) vibration; c) "near field" impulse methods; and d) wave propagation methods. Output responses are measured on the surface or with depth below the surface. Surface measurements are made with a) geophones which sense the velocity of motion; b) accelerometers; and c) linear voltage differential transformers (LVDT) which measure displacement. Measurements below the surface are made with all of the same sensors but the loading methods may include moving traffic.

Static or slowly moving loads are applied by the Benkelman Beam, LaCroix Deflectographe, and the Curviameter. Vibratory loads are applied by the Dynaflect, the Road Rater, the Corps of Engineers 71 kN (16-kip) Vibrator, and the Federal Highway Administration's Cox Van. "Near field" impulse loads, a term which will be explained subsequently, are applied by the Dynatest, KUAB, and Phoenix Falling Weight Deflectometers. "Far field" impulse loads are applied by the impact devices used in the Spectral Analysis of Surface Waves technique. Wave propagation is used by the Shell Vibrator which loads the pavement harmonically and sets up standing surface waves, the peaks and nodes of which are found by using moveable sensors. ~~It is not the purpose of this paper to recount the~~ details of construction or operation of these devices but rather to explore the methods used to analyze the data they produce.

The terms "near field" and "far field" refer to the behavior of the surface of the pavement where the measurements are made. The "near field" is within the deflection basin around the load that is applied. Surface deflections are made up of two components: the vertical deflections due to the load and the propagation of waves laterally across the surface. The "far field" is outside of the deflection basin where the surface motion is principally due to wave propagation. The boundary between the two occurs roughly at a wave number of 3.3 m^{-1} (1 ft^{-1}). The wave number, n , is defined as:

$$n = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} \quad (\text{units } \text{L}^{-1}) \quad (1)$$

where: λ = the wavelength,
 f = the frequency, and
 v = the velocity of propagation.

The distinction between "near field" and "far field" surface motion is determined primarily by the size of the deflected basin under a design wheel load, because the behavior of the materials beneath the load is different from that in the far field. The upper pavement layers in the near field are in tension due to the imposed curvature of the surface, and all layers are in an elevated level of stress. In the far field, the stresses are very small being due principally to the wave motion. These distinctions between the near and far field are illustrated in Figure 1.

Taking the diameter of a deflection basin under a typical wheel load as being equal to the critical wavelength, and using the velocity of compressive waves in typical pavement materials, the critical wave frequencies are roughly as shown in the following table.

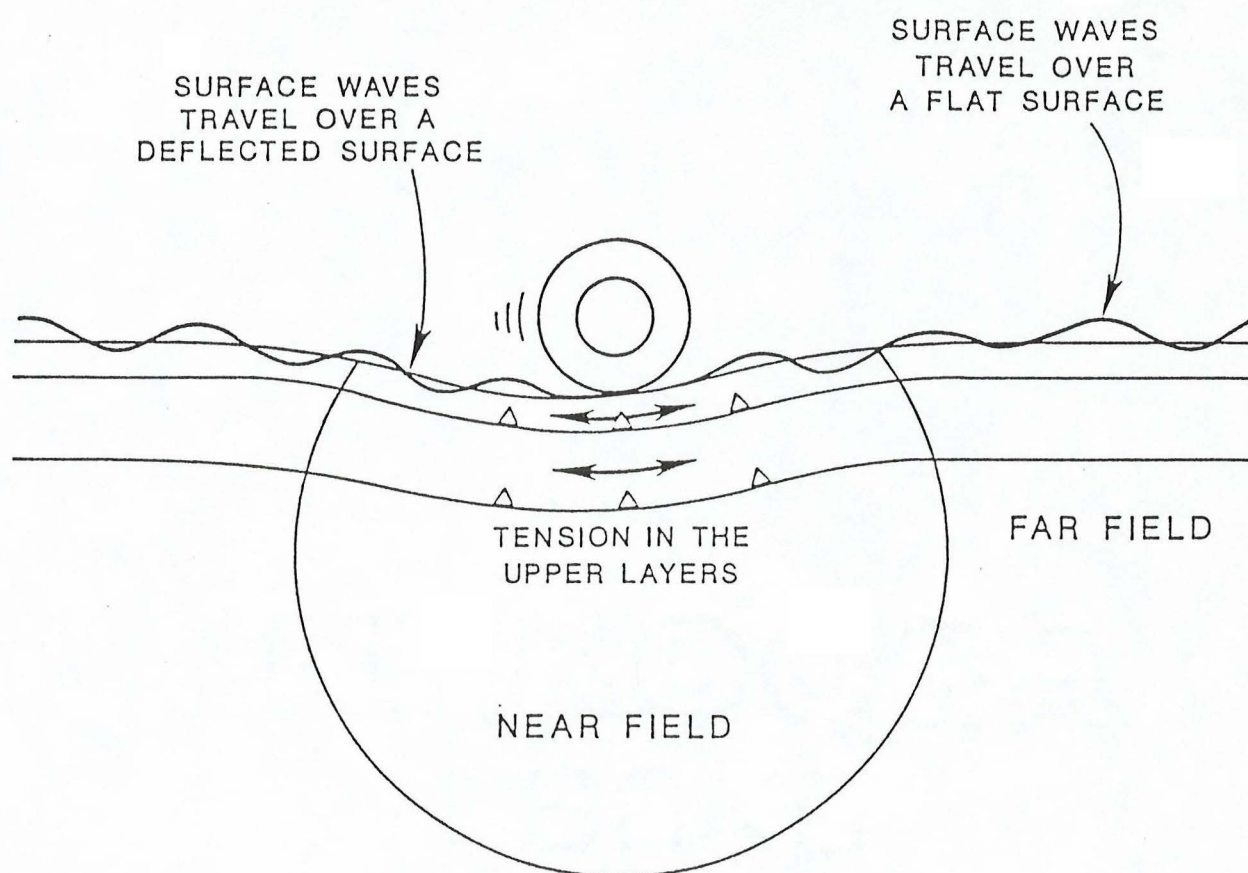


Figure 1. Schematic illustration of near field and far field characteristics.

Table 1. Pavement Material Properties and Critical Frequencies.

Material	Range of Elastic Modulus (kPa)*	Velocity of Compressive Waves (m/sec)*	Critical Wave Frequency (Hertz)
Asphalt Concrete	$1 \times 10^6 - 7 \times 10^6$	730-1700	200-500
Aggregate Base Course	$3 \times 10^5 - 1 \times 10^6$	380-760	120-250
Subgrade	$2 \times 10^4 - 3 \times 10^5$	90-350	30-110

It is important to make these distinctions among measurement devices at the outset for it is the properties of the pavement layer materials in the near field that are of interest to the pavement analyst and pavement designer. Far field measurement methods, in order to be useful for the same purposes, must be able to provide corrections from the high frequencies above about 2000 Hertz, and low stress levels which they use to the low frequencies, below about 200 Hertz, and high stress levels that are useful in pavement analysis and design.

This paper will make use of the distinction between the near field and far field, and will concentrate upon the analysis methods that are of primary interest to this symposium, that of the low frequency near field measurement methods.

ANALYSIS METHODS

The categories of analysis methods that are covered in this paper are as follows: a) historical methods; b) microcomputer methods; c) systems identification methods; and d) impulse methods for near field measurements.

Historical Analysis Methods

The first of the backcalculation methods was a closed-form solution for two layers developed by F.H. Scrivner (1). He assumed that the Poisson's ratio of each layer is 0.5. The equation he used was developed from Burmisters's equations (2), and is:

$$\frac{4\pi wr}{3p} E_1 = 1 + \int_0^{10\frac{r}{h}} (v - 1) J_0(x) dx \quad (2)$$

where: w = the surface deflection at a radial distance, r ,
from the applied load, p ,
 E_1 = the elastic modulus of the surface layer,
 h = the thickness of the surface layer,
 $J_0(x)$ = the Bessel Junction of 0-the order,
 x = $\frac{m r}{h}$, and m is a continuous variable of
integration,
 v = $\frac{1 + 4Ne^{-2m} - N^2 e^{-4m}}{1 - 2N(1 + 2m)e^{-2m} + N^2 e^{-4m}}$
 N = $\frac{E_1 - E_2}{E_1 + E_2}$

Scrivner found that the ratio $w_1 r_1 / w_2 r_2$ was useful in analyzing Dynaflect deflection basins, and developed a graph which illustrates the full range of values of the solutions to the equation given above. The graph is shown in Figure 2. As seen in both the Figure and in Equation 2, the deflection ratio $w_1 r_1 / w_2 r_2$ is a function of the modulus ratio alone, but in fact, two different modulus ratios will result in the same deflection ratio. It is up to the pavement analyst to select which is the more

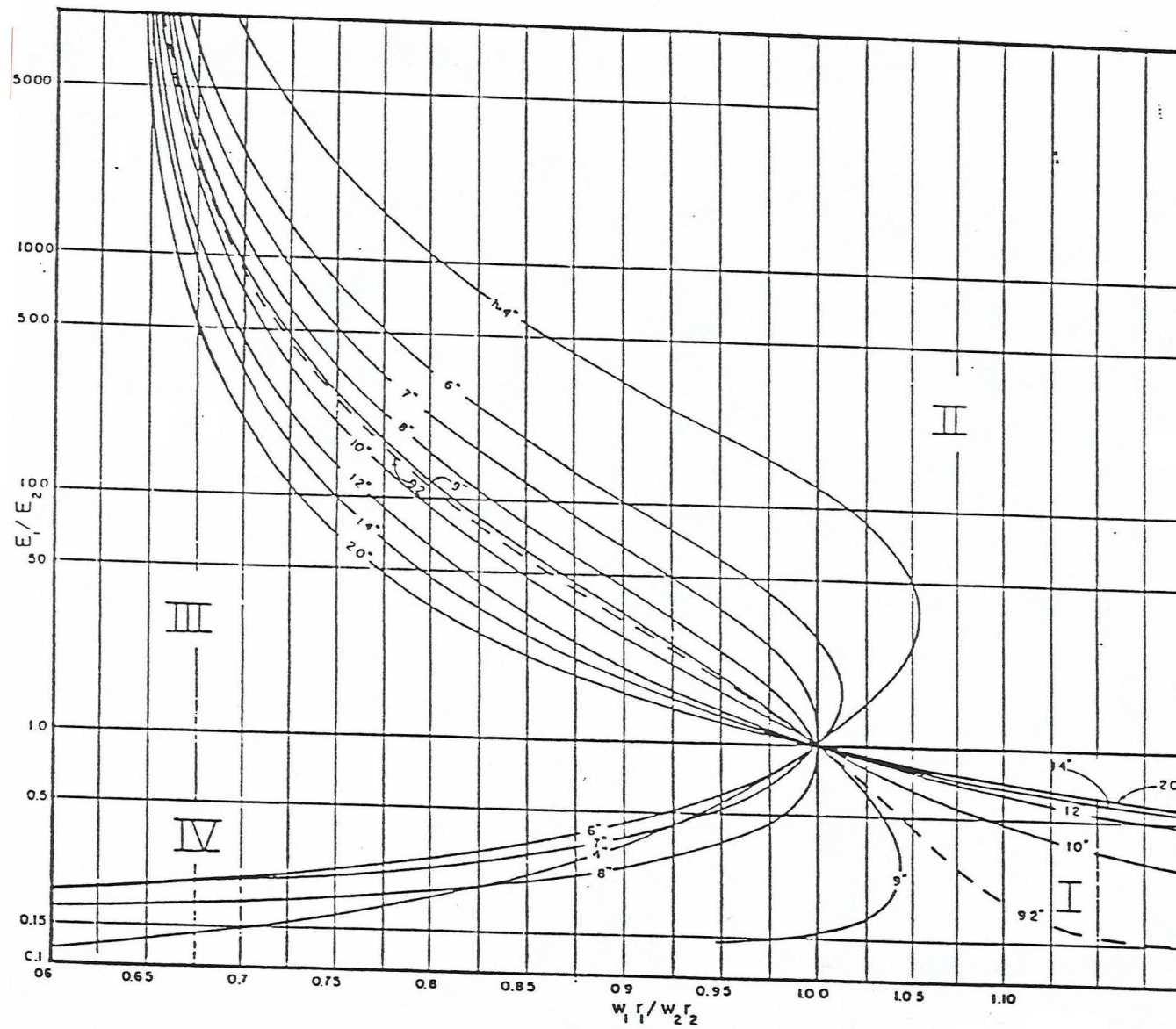


Figure 2. Graph of surface deflection ratios for two-layered elastic pavement (Ref. 1).

reasonable value of the modulus ratio: the soft-on-top solution or the stiff-on-top solution.

This graph illustrates an important point that pervades all methods of backcalculating layer moduli: an experienced analyst is always needed to guide the analysis toward the most correct solution. In this case, there are two solutions and the analyst must pick between them. In other methods, where more layers are involved, the search for the layer moduli requires finding a minimum point in an error surface, in which local minima are common. Here, the analyst must select the minimum that represents the most appropriate set of moduli for each layer.

Graphical Two-Layer Solutions

An associate of F.H. Scrivner's at the Texas Transportation Institute, G. Swift, developed a method of determining the moduli of a two-layer pavement graphically, as is illustrated in Figures 3 and 4(3). Figure 3 shows the basic chart in which the non-dimensional quantities r/h and $w/E_2/P$ are plotted for different ratios of E_1/E_2 . Once more, it is seen that the same basin shape, with deflections measured at distances from the load greater than $r = h$, can produce two different values of the modulus ratio and some expert knowledge of the expected results aids in determining which is the correct solution. The graphical method works by plotting the measured deflection basin on an overlay such as is illustrated in Figure 4, where the vertical line where the radius equals the depth of the surface layer is laid over the line in Figure 3 where the r/h ratio equals 1.0. The plotted basin is then moved up and down vertically until its general trend matches that of one of the plotted family of curves. This fit indicates the proper value of E_1/E_2 , and the

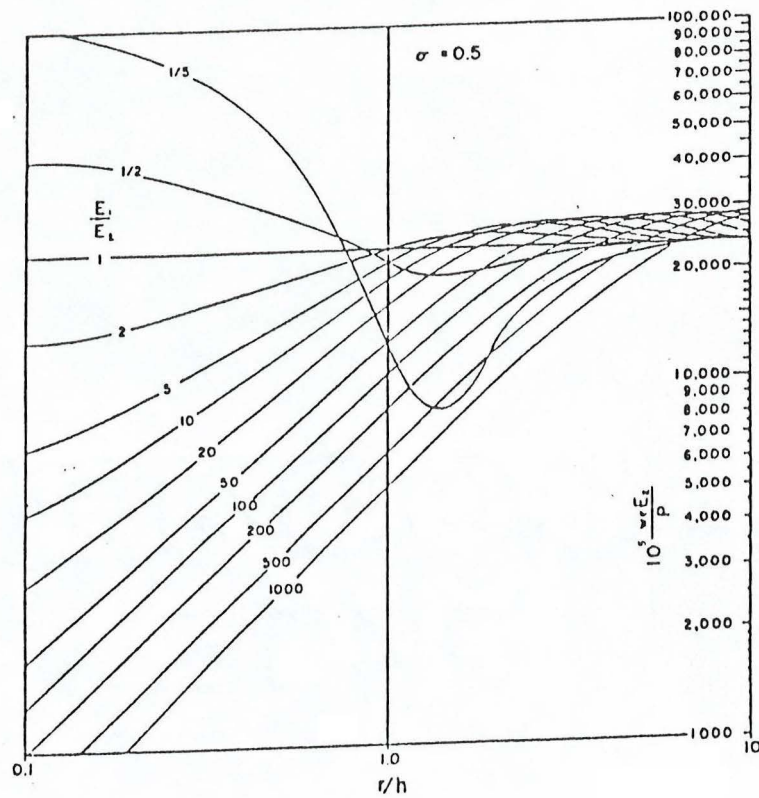


Figure 3. Graph of families of curves for different values of E_1/E_2 for two-layered pavements.

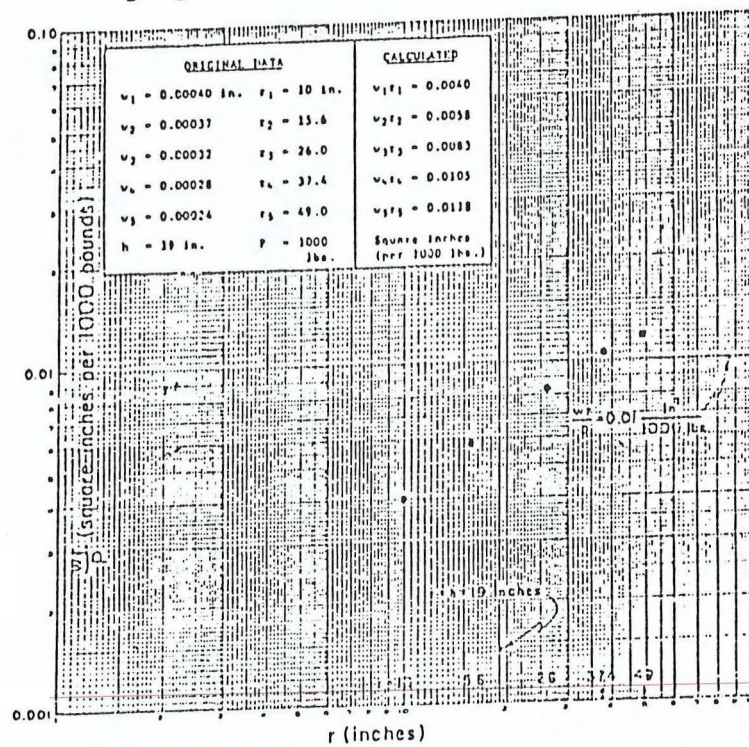


Figure 4. Plot of a measured deflection basin on the two-layered graphical overlay.

horizontal line where $wr/p = 0.01$ indicates the correct value of wrE_2/P . The values of E_1 and E_2 then can be determined from these two numbers.

"Empirical" Two-Layer Solution

Swift also developed an equation, which he called "empirical," but which has an uncanny ability to fit measured or calculated basins on two-layered pavements (4). The equation is:

$$w = \frac{3P}{4\pi} \cdot \frac{1}{r} \cdot \left[\frac{1}{E_1} + \left(\frac{1}{E_2} - \frac{1}{E_1} \right) \left(\frac{r}{L} + \frac{rx^2}{2L^3} + \frac{3rx^4}{2L^5} \right) \right] \quad (3)$$

where: $L^2 = r^2 + x^2$, and

$$x^2 = 4h^2 \left[\frac{E_1 + 2E_2}{3E_2} \right]^{2/3}$$

The interesting feature of this "empirical" equation is that the modulus ratio E_1/E_2 , is embedded in the quantity (x^2) , and the solution for (x^2) in Equation (2) is quadratic, giving two solutions just as with the graphical and closed-form solutions presented previously. Once more, this established the fact, which is well-known to experienced pavement analysts, that the search for the correct value of the modulus ratio must be guided by some expert knowledge of the desired result.

Closed-Form Multi-Layered Solution

The first closed-form, multi-layer solution for the backcalculation of layer moduli was developed at the University of Utah in a doctoral dissertation by Yih Hou (5). The central feature of this method was the least squares method used for searching for the set of moduli which will

reduce the sum of the squared differences between the calculated and measured deflections to a minimum.

In order to converge upon the set of elastic moduli for the layers that minimize the sum of squared errors, it was necessary to develop closed form expressions for the partial derivatives of the deflections, f_i , at the different radii, r_i , with respect to each of the layer moduli, E_j . The greatest amount of computer time is used in evaluating these derivatives with each successive iteration. The search method takes the form:

$$F_k^T F_k d_k = F_k^T r_k \quad (4)$$

where: F_k = the k^{th} iteration of the matrix of partial derivatives $\partial f_i / \partial E_j$, where $i = 1$ to n and n is the number of deflections measured and $j = 1$ to m where m is the number of layers on the pavement. Thus, the matrix, F_k , has n rows and m columns,

F_k^T = the transpose of the F_k matrix,

d_k = the k^{th} difference vector which is made up of the differences, $E_{j,k+1} - E_{j,k}$, between the elastic moduli, $E_{j,k}$, used in the F_k -matrix and the new moduli, $E_{j,k+1}$, which will be the new estimates of the layer moduli for the $(k + 1)$ st iteration. Thus, $d_{j,k}$
 $= E_{j,k+1} - E_{j,k}$.

r_k = the residual vector of differences between the most recently calculated surface deflections, f_i , and the measured deflections, w_i . Thus, $r_i = f_i - w_i$.

In this dissertation, Yih Hou derived and presented all of the expressions for the partial derivatives that were needed to make up the F-matrix. He states the results of proofs in References 6 and 7 which state that if the polynomial,

$$\sum_i (F_{ij} d_j - r_i)^2$$

satisfies certain conditions (i.e., having a positive Hessian matrix), then the error surface of which it contains the partial derivatives is convex, and the difference vector, \underline{d}_k , will converge to a global minimum, rather than simply to a local minimum. This statement is made on the basis of mathematical proofs found in the references. No numerical experiments were made to confirm this statement as it applies to the backcalculation problem. As noted previously, and illustrated in Figure 3 for a two-layer pavement with deflection measurements at radii beyond the thickness of the surface layer, it is possible to have two identical surface deflection basins which represent two completely different sets of layer moduli. Thus, Yih Hou's statement is correct only if it is understood that there is actually more than one global minimum. This is analogous to saying that with a quadratic equation, there are two unique roots, each being a true root of the equation. It is suggested here that as long as deflection basins are not measured very close to the load, there will be as many correct sets of moduli that fit the measured basin as there are layers in the pavement. The set of moduli to which a search method converges will depend upon the initial estimates that are used.

Equivalent Layer Methods

All equivalent methods make use of Odemark's assumption (8) which was developed for the purpose of estimating surface deflections of multi-layered pavements. Odemark's assumption is that the deflections of a multi-layered pavement with moduli, E_i , and layer thicknesses, h_i , may be represented by a single layer thickness, H , and a single modulus, E_0 , if the thickness is chosen to be:

$$H = \sum_{i=1}^m C h_i \left(\frac{E_i}{E_0} \right)^{1/3} \quad (5)$$

where: C = a constant around 0.8 to 0.9.

This very useful assumption makes it possible to use Boussinesq theory for a one-layer halfspace to estimate stresses, strains, and displacements in the halfspace which are assumed to occur in the real multi-layered pavement at the same radius, and at the depth corresponding to the transformed depth where they were calculated.

Two of the equivalent layer methods are mentioned here primarily because of the capabilities they illustrate. The first to be noted is the method developed by Ullidtz (9), which permits the use of a stress-softening nonlinear stress-strain relation in the subgrade. Calculations of rutting and fatigue life of test pavements, using strains and deflections computed using this method, have proven to be realistic (10). Backcalculation of layer moduli also appears to give reasonable results for pavements in which the layers decrease in stiffness with depth.

The other equivalent layer method to be mentioned here is that developed by Lytton et al. (11), which uses a more general form of Odemark's assumption to convert a multi-layered pavement into a single

layer above a rigid base. Instead of the exponent of the modulus ratio (E_1/E_0) being 1/3 as in Odemark's assumption, a power, n , is used and is found by nonlinear regression analysis to depend upon the thickness of the upper stiff layers. The deflection data used for this analysis were measured using a Dynaflect to provide the load, and horizontal and vertical accelerometers to measure the displacement vectors at points on the pavement surface and with depth on each of the 27 pavement sections at the TTI Pavement Test Facility (12). In this way, some nonlinearity is introduced into the backcalculation of moduli. In addition, the form of the equation of the surface deflections was taken from original work by Vlasov and Leont'ev (13) and modified by Lytton et al.(11), to give the following expression for the vertical deflection:

$$w(r, \bar{z}) = \frac{CP(1 + \nu_o)(2m + 1)}{\pi E_o H'} K_o(\alpha r) \cdot \left[\frac{H' - \bar{z}}{H'} \right]^n \quad (6)$$

$$\text{where: } \alpha = \frac{mB}{H'} \left[\frac{2(2mb + 1)}{(2mB - 1)(1 - \nu_o)} \right]^{1/2} \quad (7)$$

H' = the transformed depth of the multi-layered pavement above a rigid layer,

\bar{z} = the transformed depth of a point below the pavement surface where the vertical deflection is calculated,

$K_o(\alpha r)$ = the Bessel Function of the second kind, order zero, with argument, αr ,

B, C, m, n, H' = empirical constants to be found by nonlinear

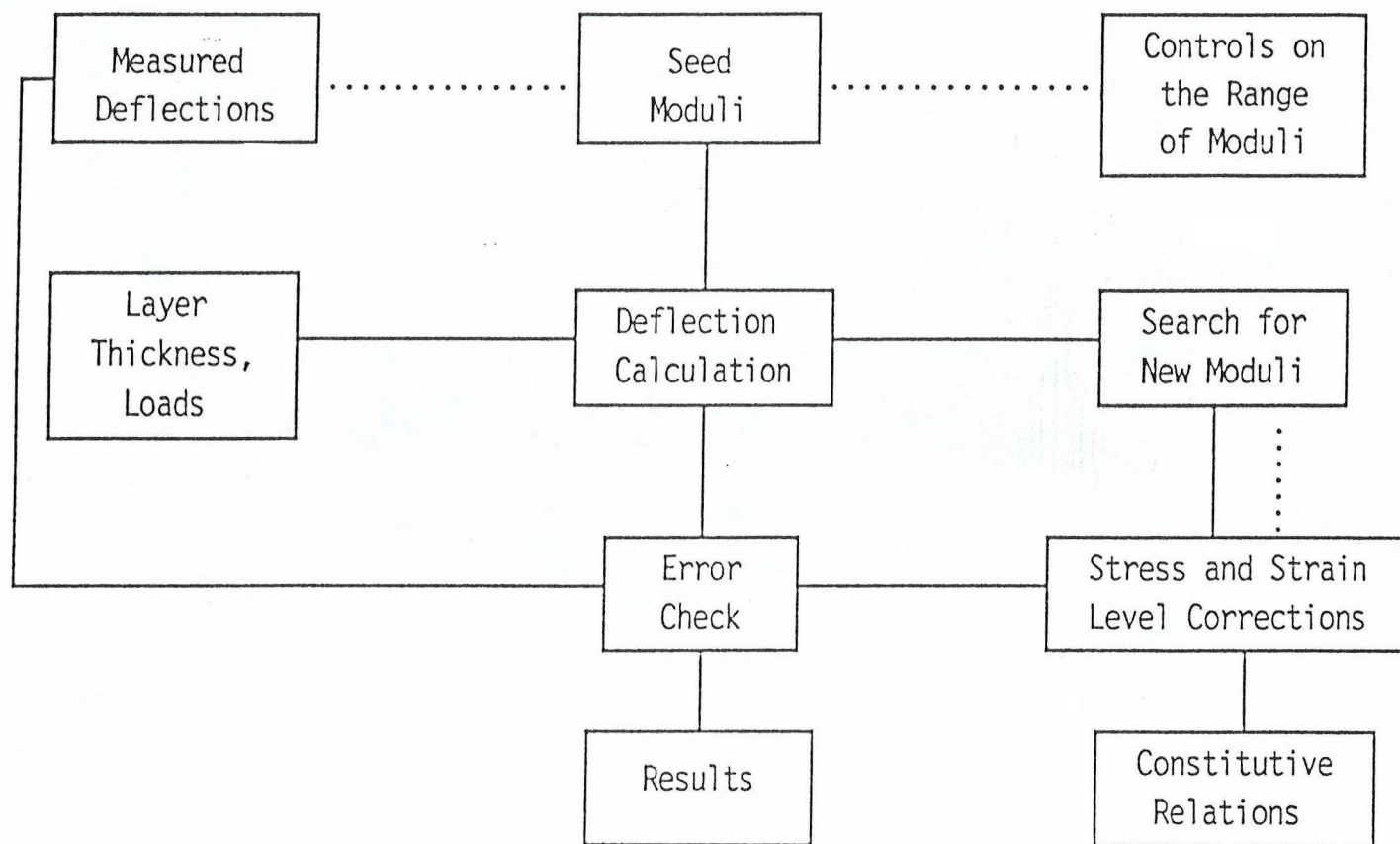
regression analysis on the measured deflection patterns on each pavement section.

Starting values of the constants B, C, m, n, and H' are 1, 1, 1, 1/3, and 1.78 m. (70 inches), respectively. The facts that the constants are all determined from field measurements, that the form of the equation fits the deflection basin, and that the calculations are all algebraic makes iterative computations very rapid on a computer. Experimentation showed that a good set of moduli can be determined in about one-thousandth of the computer time that is required for backcalculation using a layered-elastic computer program to calculate the sequential estimates of the deflection basins.

The equivalent layer methods are worthy of note for their simplicity, speed of calculation, ability to include nonlinearity, and relative accuracy for specific classes of pavements.

Microcomputer Methods

Numerous microcomputer methods have been developed to backcalculate layer moduli for pavements with three or more layers. It is impossible in the limited space of this paper to present a comprehensive summary of all of the different methods that have been developed or even those that are presented in this symposium. Instead, what is presented is a picture of what they have in common and examples of some of them which have unusual features. The features which all methods have in common are illustrated in Figure 5.



Transfer of Information or Directions:

..... occasional
 ————— usual

Figure 5. Common features of all microcomputer methods.

1. Measured Deflections. These are the surface deflections and the distances from the load at which they were measured.
2. Layer Thickness and Load These describe the pavement that is tested, the load level, and the area over which it is applied.
3. Seed Moduli. These are the starting or assumed initial values of the layer moduli. In some methods, these are either generated from the measured deflections, or from regression equations, or they are presumed values. Assumed values of Poisson's ratios are used in all methods.
4. Deflection Calculation. A number of layered elastic computer programs are used here. Some of the programs used are: BISAR, CHEVRON, ELSYM5, and others. The program takes the layer thicknesses, load, the latest set of layer moduli, and the radii to the deflection sensors and calculates the surface deflections at each. In those methods where adjustments are made for nonlinearity, stresses or strains at selected locations are also calculated.
5. Error Check. Several types of error check are used including the sum of the squared differences between the measured and calculated deflections, the sum of the absolute differences, and the sum of the squared relative errors, in which the difference is divided by the measured deflection before the ratio is squared and summed. If the error check indicates convergence within acceptable levels of tolerance, the results are printed out. ~~If not, a new iteration is started.~~
6. Results. These usually include the measured and calculated

deflections, the differences, and percent differences, the final set of layer moduli, and the error sums.

7. Constitutive Relations. These vary widely from linear elastic theory with no corrections for nonlinearity to various forms of assumed relations between the stress or strain beneath the load to the modulus of the layer. Constitutive equations may vary from layer to layer, with fine-grained soils becoming less stiff and coarse-grained soils becoming stiffer with increased levels of mean principal stress, deviator stress, horizontal stress, or some stress invariant.
8. Stress and Strain Level Corrections. These make use of the constitutive equations for each layer and any calculated stresses or strains to estimate new layer moduli to try on the next iteration. There may be interaction with the search method (Item 9) in determining the next set of layer moduli to try.
9. Search for New Moduli. This is one of the major distinguishing features of all of the microcomputer methods. The error criterion (least squares, absolute, least square relative), when plotted against the layer moduli in multi-dimensional space will form an "error surface" which may have local minima and several global minima, depending upon the radii where deflections were measured, the number of layers, the error criterion, and the degree of nonlinearity introduced by the constitutive equations and corrections. ~~The search methods attempt, by using efficient multi-dimensional search techniques, to find a global minimum which represents the least error, the best fit of the measured~~

basin, and the best set of layer moduli. As noted in the previous discussion of this point, it is advisable not to assert that the set of moduli derived from any search is the **only** set of moduli possible without having mathematical, rather than empirical, proof of the point.

10. Controls of the Range of Moduli. In order to guide the iterative search toward a set of moduli that are considered to be acceptable, numerous controls are programmed to direct the search away from unwanted or unreasonable values of the moduli. In most cases, the controls make some assumption of the type of pavement that is analyzed, assuming, for example, that the moduli decrease with depth, or that the subgrade modulus is constant with depth, or that a rigid layer exists at a depth below the subgrade, or that a relationship exists between the modulus of the lower layers and that of the layer above it. Stabilized layers or thin, soft layers do not fit these patterns, and will cause difficulty in convergence or in the final results.

Of interest as illustrations of microcomputer backcalculation programs are the MODCOMP programs developed by Irwin (14) at Cornell University, the "___DEF" series of programs developed by Bush (15) at the U.S. Army Corps of Engineers Waterways Experiment Station, and the MODULUS program developed by Uzan, Lytton, and Germann (16) at Texas A&M University. ~~The two programs developed by Bush include the CHEVDEF and BISDEF programs, in which the deflection calculations are performed by the~~ CHEVRON program (17), and the BISAR program (18), respectively. Bush

furnished a copy of the CHEVDEF program to Texas A&M University which substituted the ELSYM5 program into it and termed it ELSDEF.

The MODCOMP program uses the CHEVRON program for deflection calculations and is notable for its extensive controls on the seed moduli and the range of acceptable moduli. Also, it uses the sum of squared differences as its error criterion. The "___DEF" program series are notable for the gradient search technique that it employs (19). The MODULUS program is notable for several features in which it departs from the usual microcomputer program pattern, as follows:

1. It handles up to four layers.
2. It can make use of any deflection calculation method, linear elastic layered or finite element.
3. It requires a calculated data base of deflections for all combinations of high, low, and medium levels of moduli for each layer. Layer thicknesses are set.
4. It uses the relative squared error as a criterion, principally because the manufacturer's specifications of the geophone are in terms of a percent error, and thus an absolute or squared difference error criterion is not consistent with it.
5. It uses interpolation to search for the initial and all subsequent sets of moduli
6. It uses the Hooke-Jeeves pattern search algorithm (20) which has been shown to converge more rapidly than a Newton search method.
7. ~~It produces answers between 30 and 100 times faster than other~~ microcomputer programs and is thus suited for production determinations of layer moduli.

8. It has been modified to use vertical deflections with depth (21) to backcalculate layer moduli from the measurements made with the Multi-Depth Deflectometer.

A paper which describes this method appears in this symposium (16).

Nonlinearity

The nonlinear relations that are known to exist between the modulus of a material and its stress state are addressed in different ways by linear elastic and by finite element methods. With the layered linear elastic programs, the modulus of the layer is adjusted based upon the stress conditions in the layer beneath the load. Because the modulus varies with the stress state, it changes continuously from what it is beneath the load to its value at a remote distance from the load. But the layered elastic methods are capable of using only one modulus for an entire layer. Consequently, the modulus that best fits the measured deflection basin is an "average" modulus that corresponds to a stress state at some distance away from the load. The distance varies with the pavement structure. Figure 6 is an illustration of this in which the moduli of the base course were backcalculated using a layered elastic program (22). Three different loading devices were used to apply the load: Dynaflect, Road Rater, and three levels of load from the Dynatest Falling Weight Deflectometer. The initial tangent modulus of the stress-strain relation of the base course layer was assumed to vary with the mean principal stress in the center of the layer immediately beneath the load. When the iterations converged to a final value of the initial tangent modulus, its relation with the mean principal stress showed that it varied

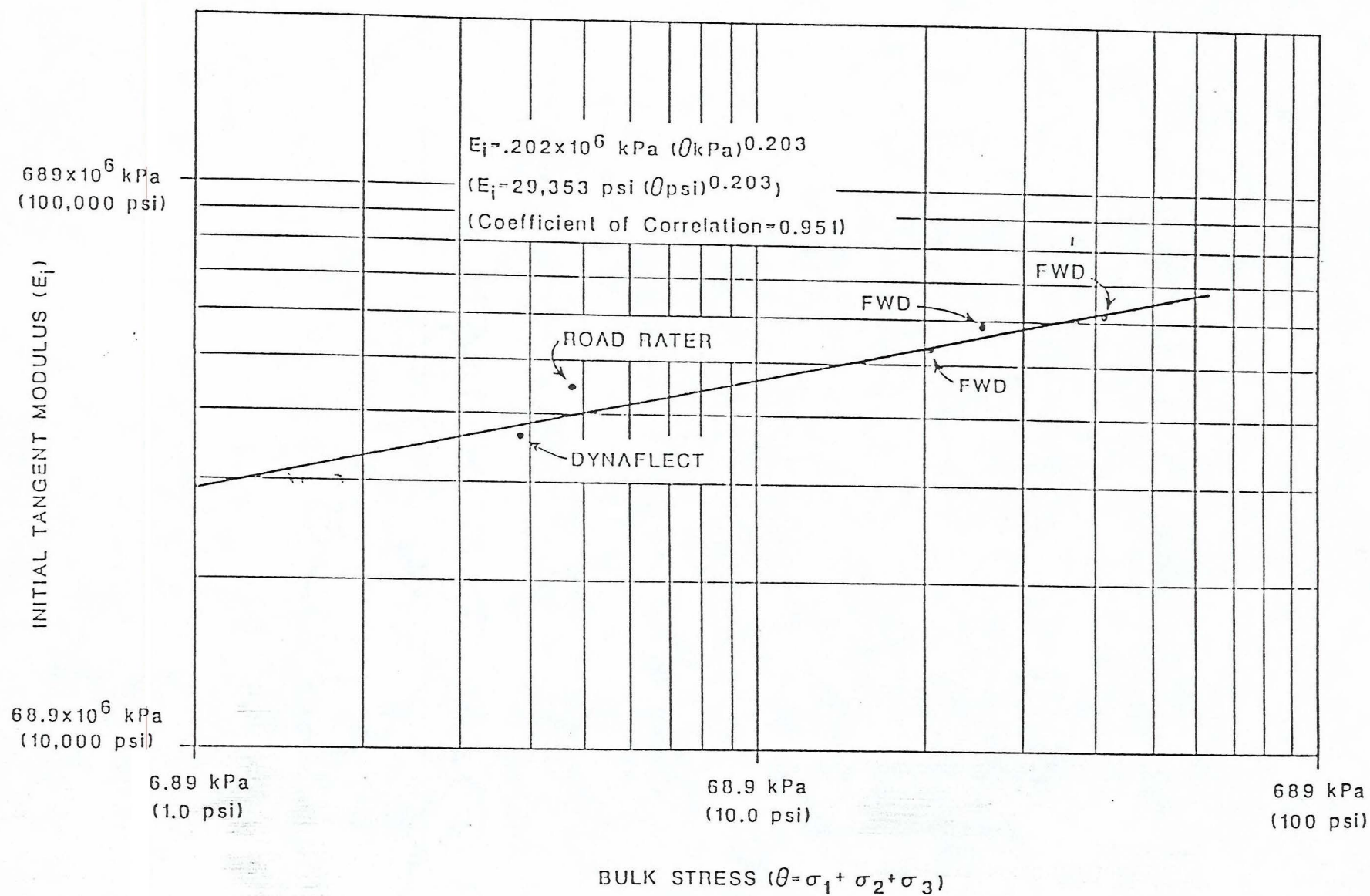


Figure 6. Initial tangent modulus versus bulk stress for crushed limestone (Ref. 22).

with the 0.203 power of that stress invariant, about half of the value that would be expected from laboratory tests. The reason for this is that an average layer modulus must be used in layered elastic programs to fit measured deflection basins. As a corollary to this, one should not use laboratory-derived constitutive equations for a layer material in fitting a deflection basin because it will introduce a systematic error in the resulting back-calculated moduli.

The only really consistent way of using laboratory-derived constitutive equations in backcalculations or, conversely, in deriving constitutive relations from field measurements at different load levels is to make use of a finite element method which permits the modulus of a layer to vary from point to point. Figure 7 is an illustration of how the modulus varies within the base course and subgrade when it is allowed to vary from point to point in a finite element program (23). As is apparent from this graph, the modulus is not constant with radius or with depth within any layer. In the figure, the base course is a stress-stiffening and the subgrade is a stress-softening material.

Another consequence of this nonlinearity is that no correlation between different pavement loading devices should be expected to be found that is independent of the pavement structure unless finite element methods are used in the backcalculation of the layer moduli. Even the excellent correlation shown in Figure 6 should be expected to change if the same measurements were made on a different pavement. And no consistent correlation between devices can be expected unless there is an agreement upon the constitutive models that will be used to represent the stress-dependency of the stress-strain relations of the layer materials.

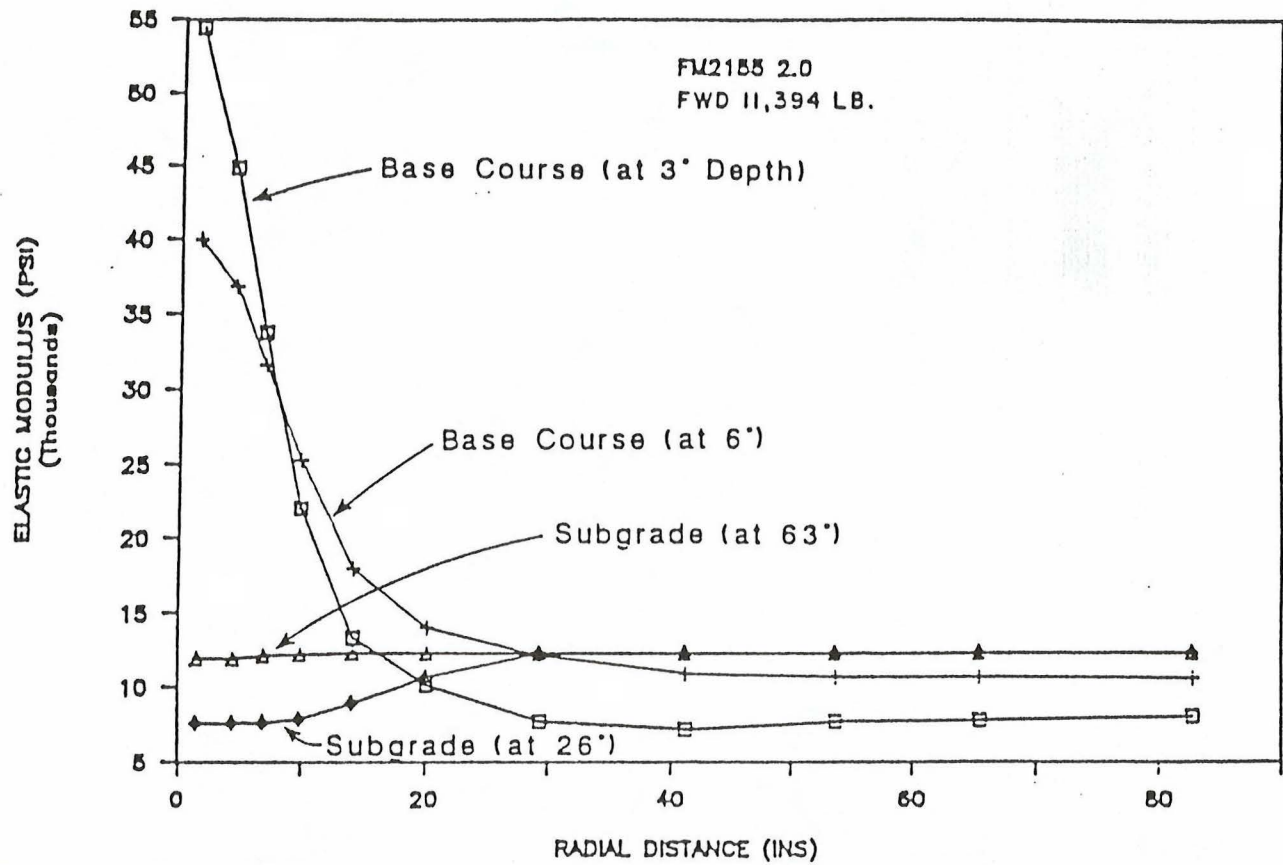


Figure 7. Nonlinear elastic modulus profiles from ILLIPAVE (Ref. 23). (1 in. = 2.54 cm.; 1 psi = 6.895 kPa).

An elasto-plastic stress-strain curve such as is illustrated in Figure 8 could serve the general purpose of representing all unbound materials in a pavement (24). The simplest form of the stress-strain relationship is:

$$\left[\frac{1 - a}{\left(\frac{E}{E_1} - a \right)} \right]^m - \left[\frac{(1 - a)\epsilon}{b} \right]^m = 1 \quad (8)$$

where: E_1 = the initial tangent modulus which is itself dependent upon the stress state,
 a = the ratio, E_p/E_1 , of the plastic modulus to the initial tangent modulus,
 b = the ratio σ_y/E_1 , of the maximum "plastic yield" stress to the initial tangent modulus,
 ϵ = the strain,
 E = the secant modulus, and m = the exponent.

The use of this constitutive relation in backcalculation is illustrated in a paper in this symposium (22). Values of a , b , and m which have been derived from torsional resonant column tests are given in Table 2. Whether these same constants will hold with repeated load triaxial tests run at loading times comparable to pavement loadings is unknown. When the ratio of the secant modulus to the initial tangent modulus, E/E_1 , is plotted against the strain level, ϵ , the resulting graph is shown schematically in Figure 9. The curve approaches the value of a asymptotically.

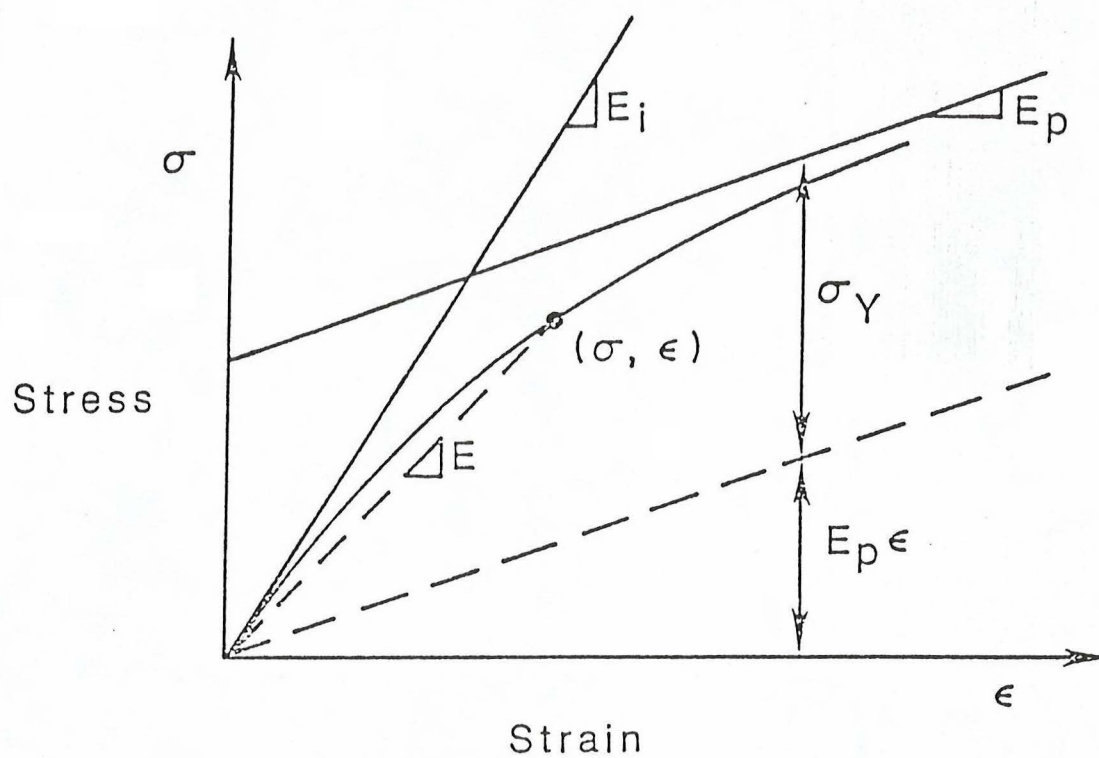


Figure 8. Schematic illustration of an elasto-plastic hyperbolic stress-strain curve (Ref. 24).

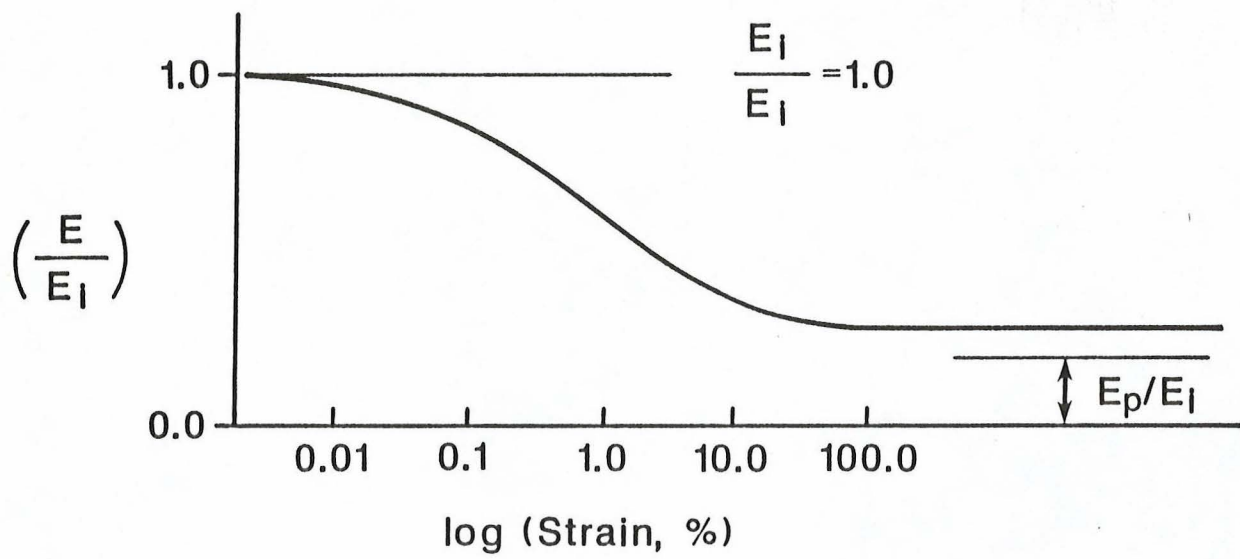


Figure 9. Graph of modulus ratio versus log strain.

Table 2. Dimensionless Constants for the Elasto-Plastic Hyperbolic Stress-Strain Curve

Type of Soil	Dimensionless Constants			Source of Stress-Strain Curve Data
	a	b	m	
Fine-Grained	0.0529	0.0435	1.002	(25)
Granular	0.0749	0.0261	0.915	(26)

The initial tangent modulus is assumed to vary with the stress state depending upon the type of material that is represented. Relations that are commonly used at present and which are based upon laboratory testing include the following:

$$E_1 = K_1 (\theta)^{K_2} \quad (9)$$

$$E_1 = K_3 (\sigma_3)^{K_4} \quad (10)$$

$$E_1 = K_5 (\sigma_d)^{K_6} \quad (11)$$

where: K_1 through K_6 = material coefficients and exponents,
 θ = the mean principal stress,
 σ_3 = the minimum principal stress, and
 σ_d = the deviator stress, i.e., the difference between the maximum and minimum principal stress.

Typical values of K_1 through K_6 are given in Table 3 for base course and subgrade materials.

The exponents in this table show that the fine-grained soils are stress-softening (negative exponent) and the coarse-grained soils are

Table 3. Typical values of Base Course and Subgrade Constants K_1 through K_6 (moduli in psi)

Material		K_1	K_2	K_3	K_4	K_5	K_6
Crushed Stone	max			15,000	0.45		
	min			5,000	0.63		
Crushed Gravel	max	25,000	0.38				
	min	7,800	0.60				
Crushed Limestone	max	11,000	0.40				
	min	2,600	0.65				
Granitic Gneiss	max	34,000	0.19				
	min	1,500	0.73				
Basalt	max	8,900	0.47				
	min	4,700	0.65				
Sand	max			13,000	0.35		
	min			6,700	0.55		
Silty Sand	max	3,100	0.37				
	min	1,900	0.61				
Clayey Sand	max					25,000	-0.80
	min					---	---
Silty Clay	max					66,000	-0.38
	min					24,000	-0.11
Lean Clay	max					27,000	-0.50
	min					---	---
Highly Plastic Clay	max					25,000	-0.77
	min					---	---

(1 psi = 6.895 kPa)

stress-stiffening (positive exponent). These nonlinear relations should be used in backcalculating layer moduli to account for the stress and strain sensitivity of the layer materials only if a finite element program is used in calculating the deflections. It should be noted that the exponents for granular base courses in Table 3 are around twice as large as the exponent found by using a layered elastic program, as in Figure 6. Current research indicates, both empirically (27) and theoretically (28), that the modulus of all materials depends upon both the mean principal stress and the deviator stresses, and acts as both stress-stiffening and -softening depending upon the relative level of these stresses.

Errors in Computing Layer Moduli

There are several sources of error in the backcalculated moduli besides the nonlinearity of the stress-strain relations of the materials in the pavement layers. These errors which are introduced by the deflection calculation model and its presumed constitutive relations are **systematic** and thus cannot be eliminated or reduced by repeated measurements or calculations. Only **random** errors can be so reduced or eliminated and the sources of these are in the measurements that are made (both force and deflection) and in the spatial variation of the materials in the layers. It will be of little value to the American Society for Testing and Materials to attempt to set standards on pavement deflection testing or on modulus backcalculation procedures without first making a careful assessment of the magnitude of both the random and systematic errors.

~~An exercise of this nature was conducted by Texas A&M as an activity of the Transportation Research Boards Committee A2B05 on the Strength and Deformation Properties of Pavements. Only a few of the major findings of~~

that exercise will be reviewed here. A total of 15 deflection basins were sent to different agencies to have layer moduli determined by backcalculation. Eight of the basins were calculated: four by the BISAR layered elastic program and four by the ILLI-PAVE cylindrical coordinate finite element program. The remaining seven basins were measured. The exact answer was known with the calculated basins and were unknown with the measured basins. All of the backcalculation procedures used some form of linear layered elastic calculations, and thus the correspondence of the backcalculated moduli with those used in the finite element program was not expected to be as good as with the basins calculated with the BISAR program.

Figure 10 shows the range of moduli determined by several of the agencies for the asphaltic concrete layer when the deflection calculations were made with the BISAR program. The asphaltic concrete layer thicknesses ranged from 2.5 to 13 cm. (1-5 in.). Figure 11 shows the range of moduli for the subgrade with the original deflections calculated by BISAR. In each case, the correct answer is the heavy line running vertically down the page. The horizontal scale is logarithmic. It is noteworthy that the range of moduli is smaller with the subgrade than with the asphaltic concrete layer, and that the correct answer was bracketed by all agencies. Nevertheless, a factor of 10 or more separates the lowest modulus from the highest modulus estimated by many of the agencies for asphaltic concrete.

Figure 12 shows the range of backcalculated moduli for the base course when the original basin was calculated with the finite element program. ~~Figure 13 gives the range of subgrade moduli that were backcalculated.~~ The heavy vertical line indicates the modulus at the center of the base course layer and at the top of the subgrade, both directly

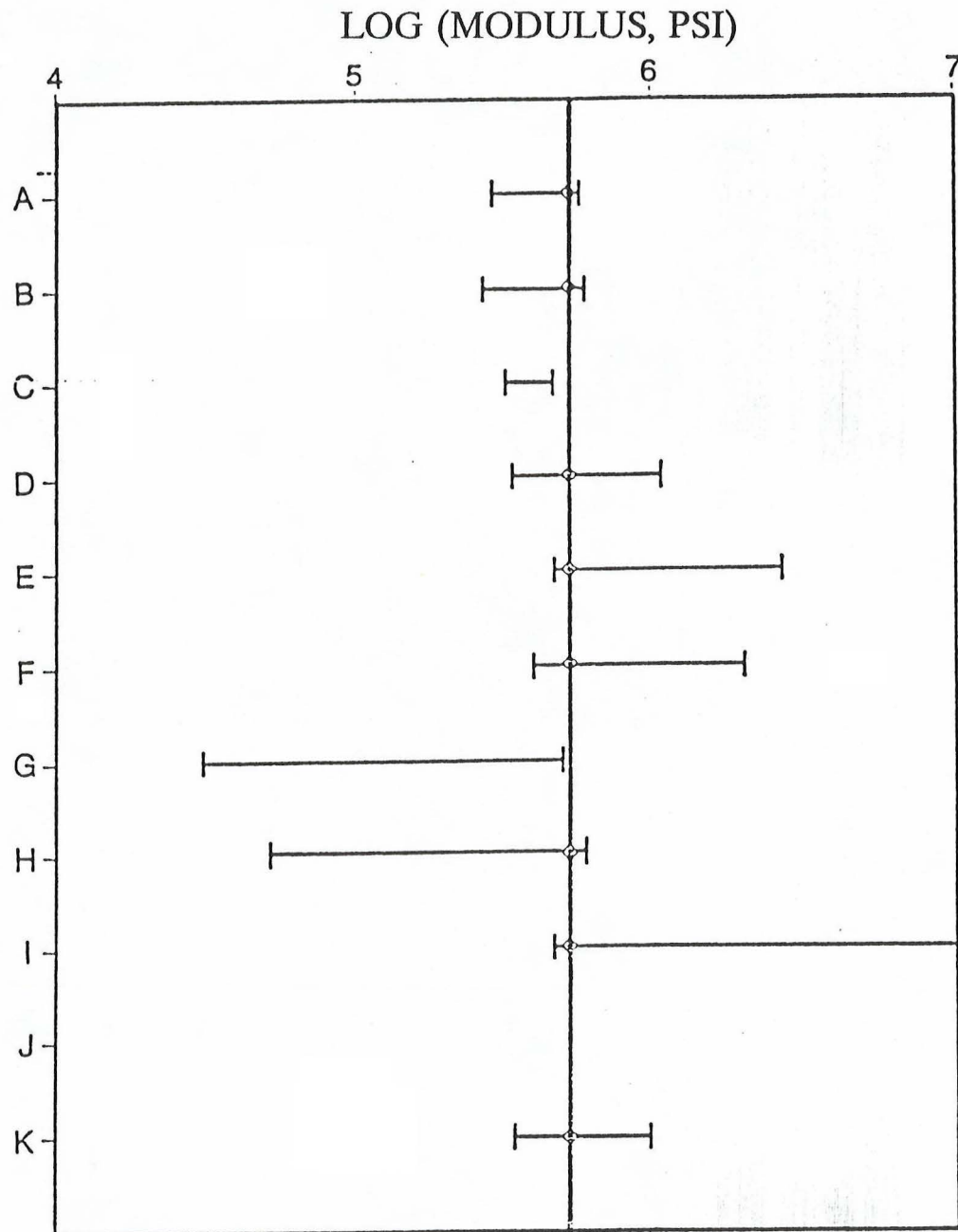


Figure 10. Range of backcalculated moduli of the asphaltic concrete layer. Original deflection calculations made with the BISAR program (1 psi = 6.895 kPa).

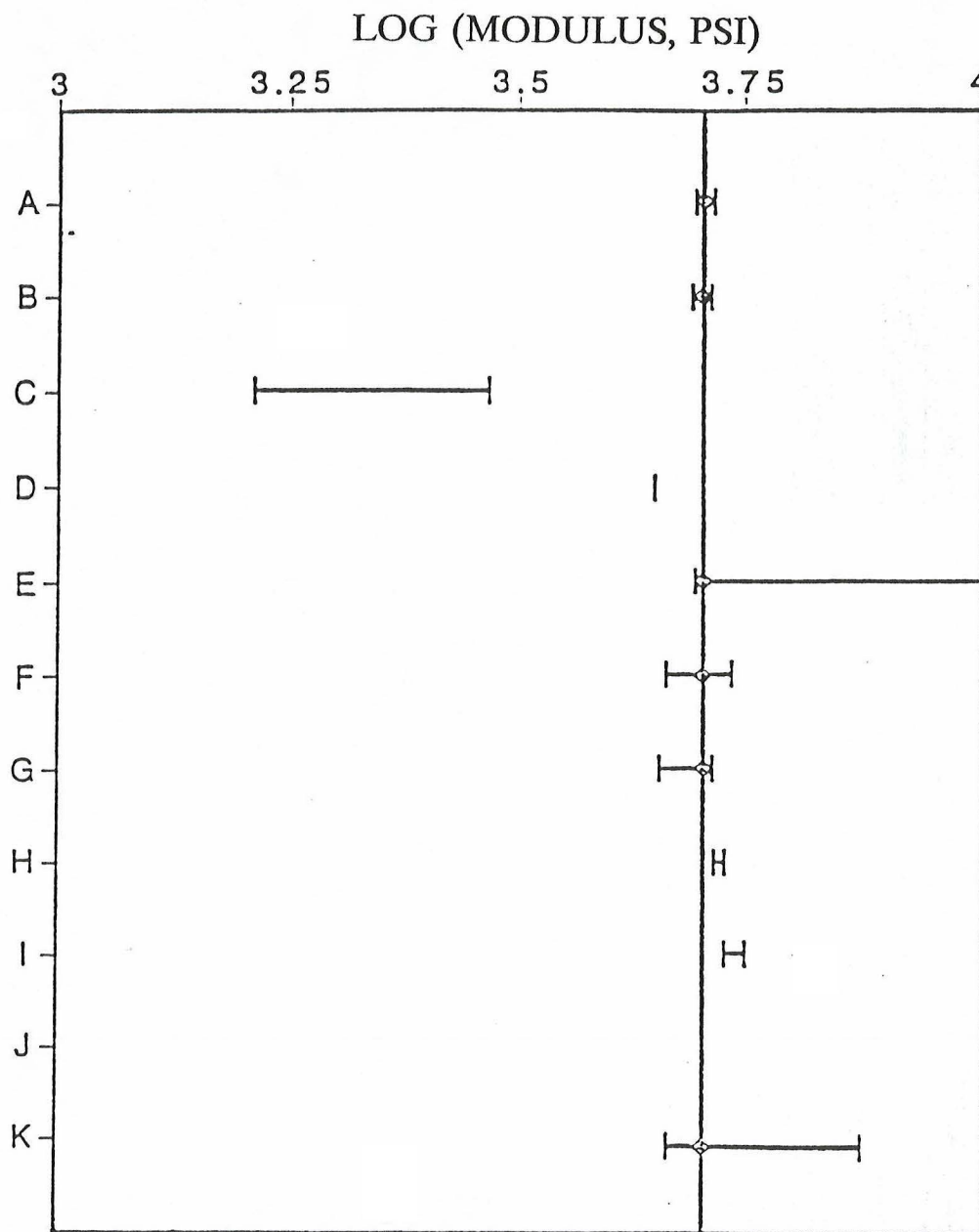


Figure 11. Range of backcalculated moduli of the subgrade. Original deflection calculations made with the BISAR program. (1 psi = 6.895 kPa).

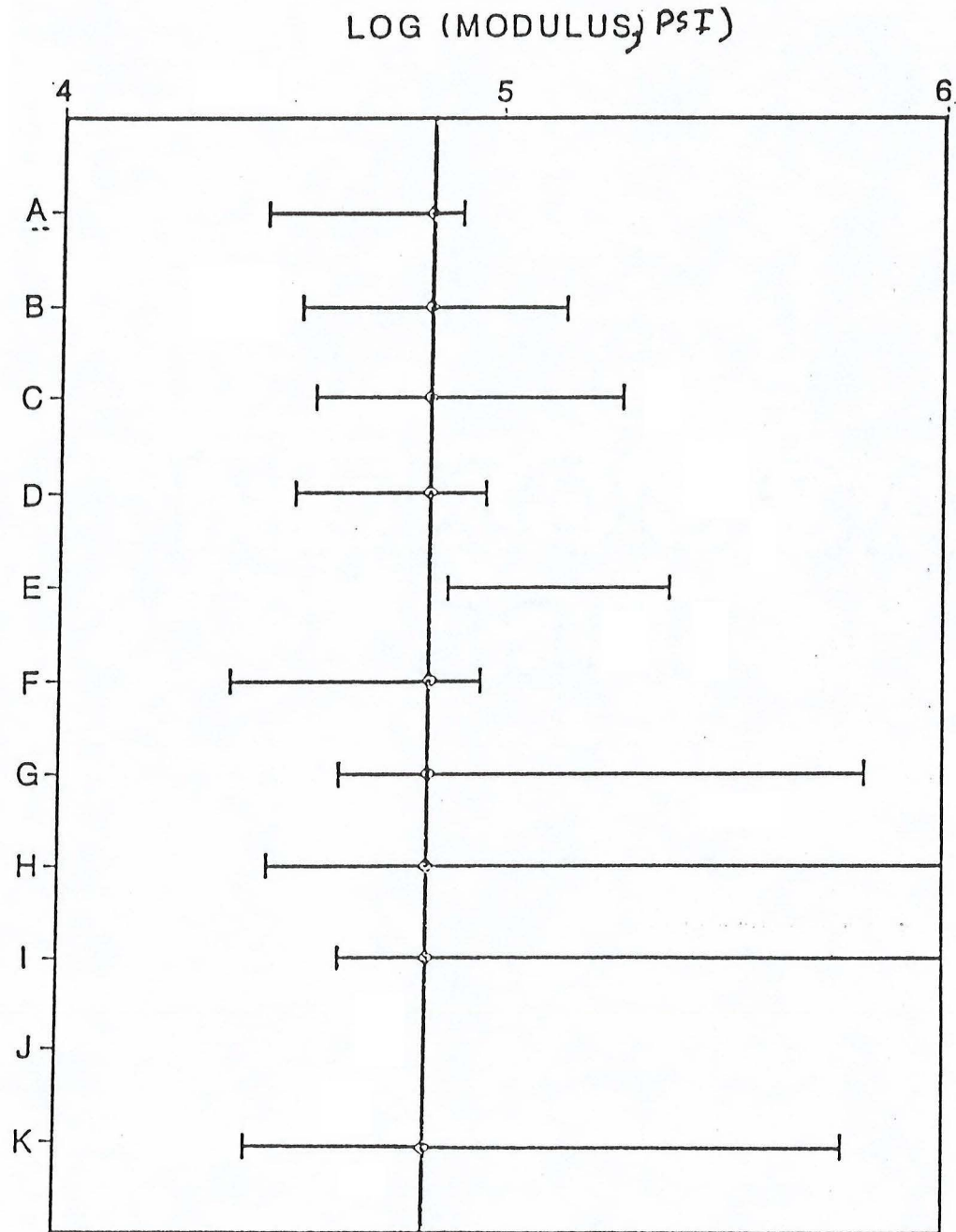


Figure 12. Range of backcalculated moduli of the base course. Original deflection calculations made with the ILLI-PAVE program. (1 psi = 6.895 kPa).

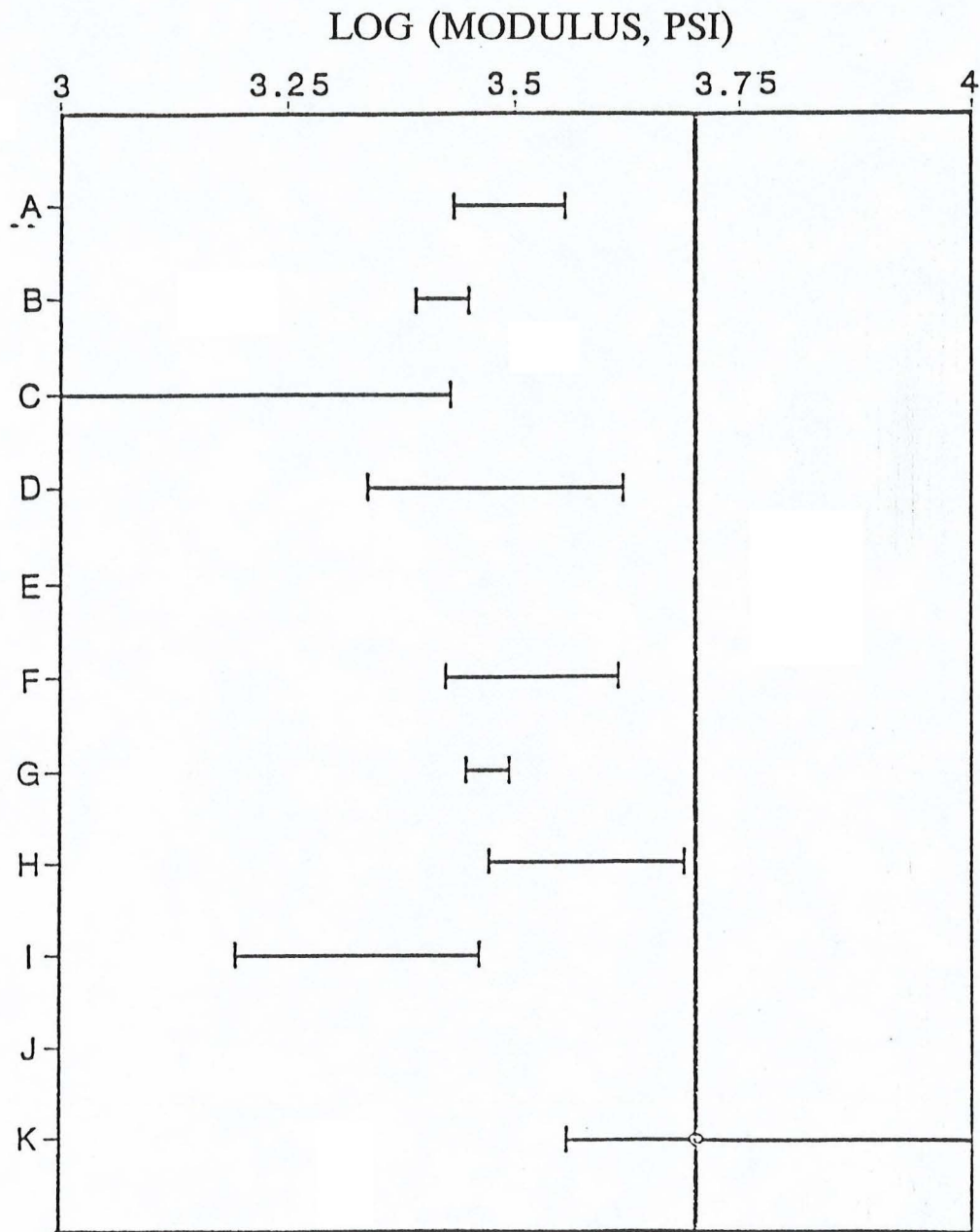


Figure 13. Range of backcalculated moduli of the subgrade. Original deflection calculations made with the ILLI-PAVE program. (1 psi = 6.895 kPa).

beneath the load. The range of moduli are greater than with the BISAR-calculated basins, as expected. In some cases, the agencies failed to bracket the correct value of the modulus.

These results give a realistic picture of the accuracy of the result that can be achieved with backcalculation methods as they are used at present. It would be misleading to claim, and unwise to expect greater accuracy than the results shown in these four figures. Further improvements in backcalculation methods are indicated.

The backcalculation of the moduli of the pavements on which deflection basins were actually measured was a test of the consistency between agencies, since the correct answer was unknown. Agency A was chosen as the datum because of its consistent accuracy with the calculated basins and the results from the other agencies were compared to it. Table 4 gives the absolute differences between Agency A and selected other agencies for all of the materials represented in the measured basins. The table shows some fairly small differences with Agencies B, C, and G, and large differences with Agencies H and I. The differences tended to be larger with the upper layers and smaller with the subgrade. Because Agency A used BISAR, the average absolute difference was computed for all agencies which also used BISAR as was the average absolute difference for all agencies. Statistical tests of significance of these differences indicated that there are significant differences between materials, between agencies, and between methods of analysis. Knowledge of the details of the analysis that was performed by each agency indicates that the difference between agencies is largely due to the experience of the analyst.

Table 4. Averaged Absolute Relative Difference of Backcalculated Modulus Compared with Agency A (Decimal).

Materials	Agencies									Avg. (BISAR)	Avg. (ALL)
	B*	C*	D	E	F*	G	H	I*	K		
Asphalt Concrete	.263	.494	6.701	1.890	2.038	.623	20.646	11.269	7.644	3.516	5.730
Cement Stab. L.S.	.091	.400	.272	.836	.586	.393	59.116	3.178	.140	1.064	7.224
Lime Stab. L.S.	.065	.098	.372	.463	.605	.530	15.055	4.101	.188	1.217	2.386
Cr. Limestone	.249	.157	1.214	.099	.424	.997	259.446	40.290	.914	10.280	33.754
Sandy Gravel	.032	.083	.428	.606	.603	.509	.452	.538	.610	.314	.429
Sandy Clay	.022	.011	.322	.358	.768	.433	.481	.740	.229	.385	.374
Plastic Clay	.044	.083	.475		.484	.393	4.915	.953	.965	.391	1.039
AVERAGE	.109	.189	1.398	.709	.787	.554	51.444	8.724	1.527	2.452	7.277

* These agencies used the BISAR layered elastic program (Ref. 18).

Pavement layer thickness ranges were as follows: surface, 2.5-13 cm. (1-5 in.); base, 10-30 cm. (4-12 in.); subbase, 10-30 cm. (4-12 in.).

Need for an Expert or an Expert System

The study of the backcalculations by different agencies brings out a point that has been known by backcalculation analysts for some time and that is that the search for the most acceptable set of moduli must be guided by a person who has experience both in analysis and with the materials and deflections in question. Backcalculation of moduli can never be automated unless a suitable expert system is constructed. The beginnings of such a system are presented in a paper in this symposium (29).

An expert or expert system is needed at six steps along the analysis process:

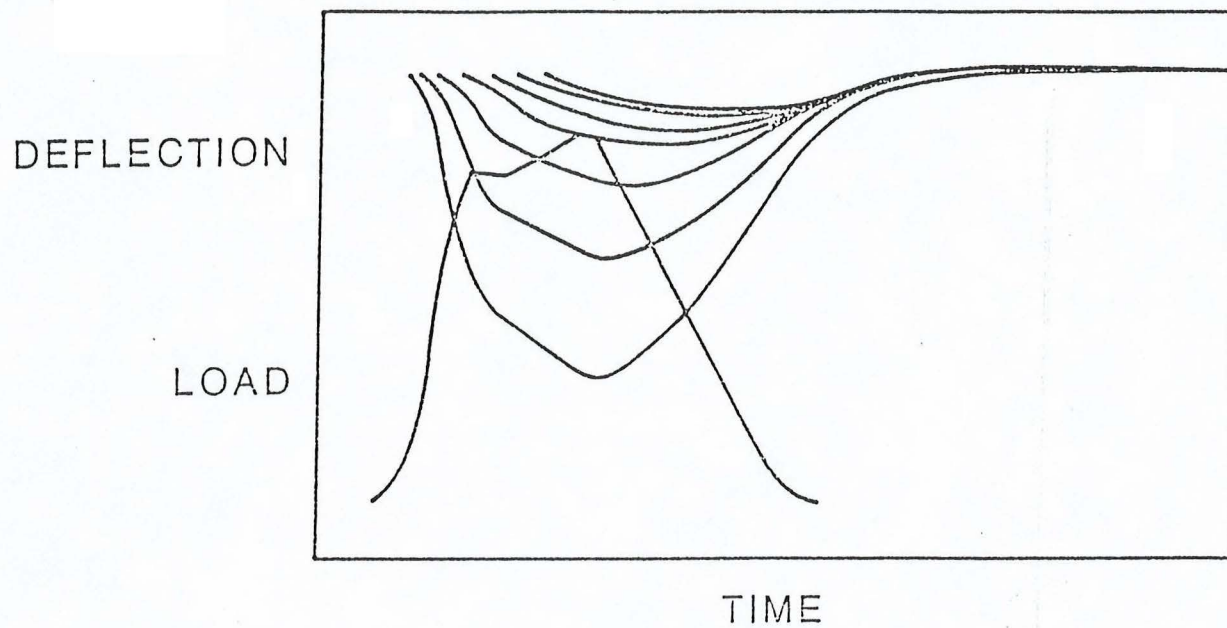
1. Selection of a method for calculating deflections,
2. Selection of constitutive relations for the materials in the pavement layers,
3. Making stress and strain level corrections,
4. Selecting seed moduli,
5. Selecting realistic closure tolerances, and
6. Interpreting anomalous results, especially when thin layers or temperature gradients are present in the pavement.

The selection of the calculation methods should be consistent with the method used in pavement design. If layered elastic calculations are used in pavement design, they should also be used in backcalculating layer moduli. Finite element methods should be used when it is desired to ~~compare constitutive relations between the laboratory and the field.~~

Impulse and Response Analysis Methods in the Near Field

When the falling weight drops to a pavement surface, an impulse enters the pavement and creates body waves and surface waves. The geophone sensors pick up the vertical velocity of the pavement surface and a single analog integration of the signal produces the deflection versus time trace. Figure 14 shows a typical set of force versus time impulses and deflection versus times responses. Each entire signal is completed within 100 milliseconds. Usually, these signals are used to extract the maximum force and the maximum deflection from each geophone and to print them out to be analyzed by elastic methods. But there is much more information in these signals than simply their maxima.

One method of tapping this additional information is to perform a Fast Fourier Transform (30) on the force-time impulse and on each deflection-time response. Such a transform requires first of all that the signal be converted from an analog to a digital signal. A normal sampling interval is 400 microseconds which permits the impulse and response signals to be broken up into 256 ordinates. The Fast Fourier Transform is a discrete Fourier transform that is made on such digital samples of a signal. The transform breaks up a signal into its component frequencies and produces a complex number for each frequency, $a(f) + ib(f)$. The magnitude of this complex number is $(a^2 + b^2)^{1/2}$ and the phase angle, ϕ , is $\arctan(b/a)$. If the transform of the deflection signal is divided, frequency by frequency, by the transform of the load impulse, the result is a transfer function, which is also a complex number and which is also a function of frequency. A graph of the magnitude and phase angle for typical transfer functions is shown in Figures 15 and 16, respectively, for the geophones which are placed 1, 3, and 5 feet (0.3, 0.9, and 1.5 m)



TYPICAL FWD LOAD IMPULSE AND
GEOPHONE RESPONSE WITH TIME

Figure 14. Typical FWD load impulse and geophone response with time.

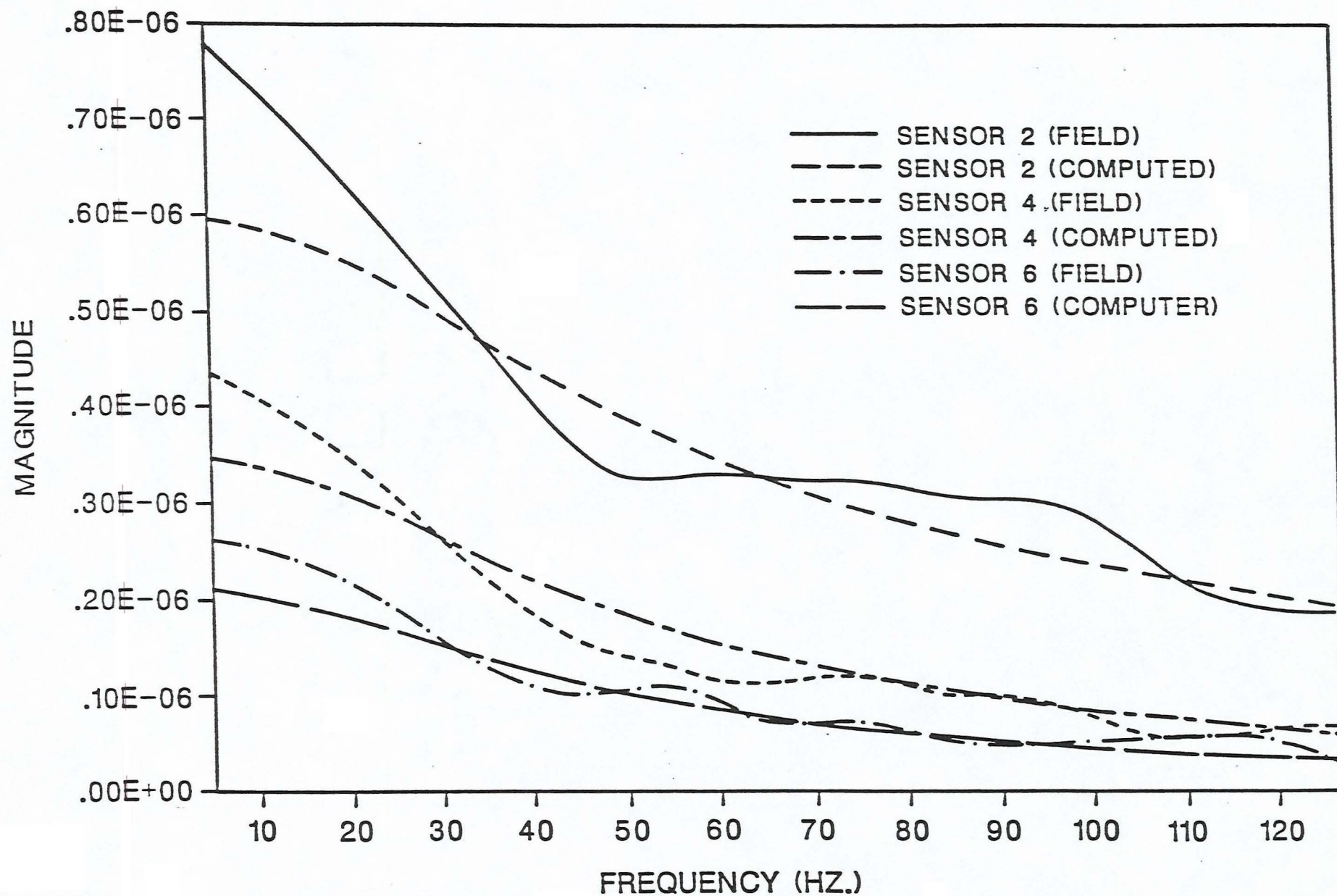


Figure 15. Graph of the magnitude of the transfer function versus frequency for geophones at radii 1, 3, and 5 feet (0.3, 0.9, and 1.5 m.) (Sensors 2, 4, and 6.)

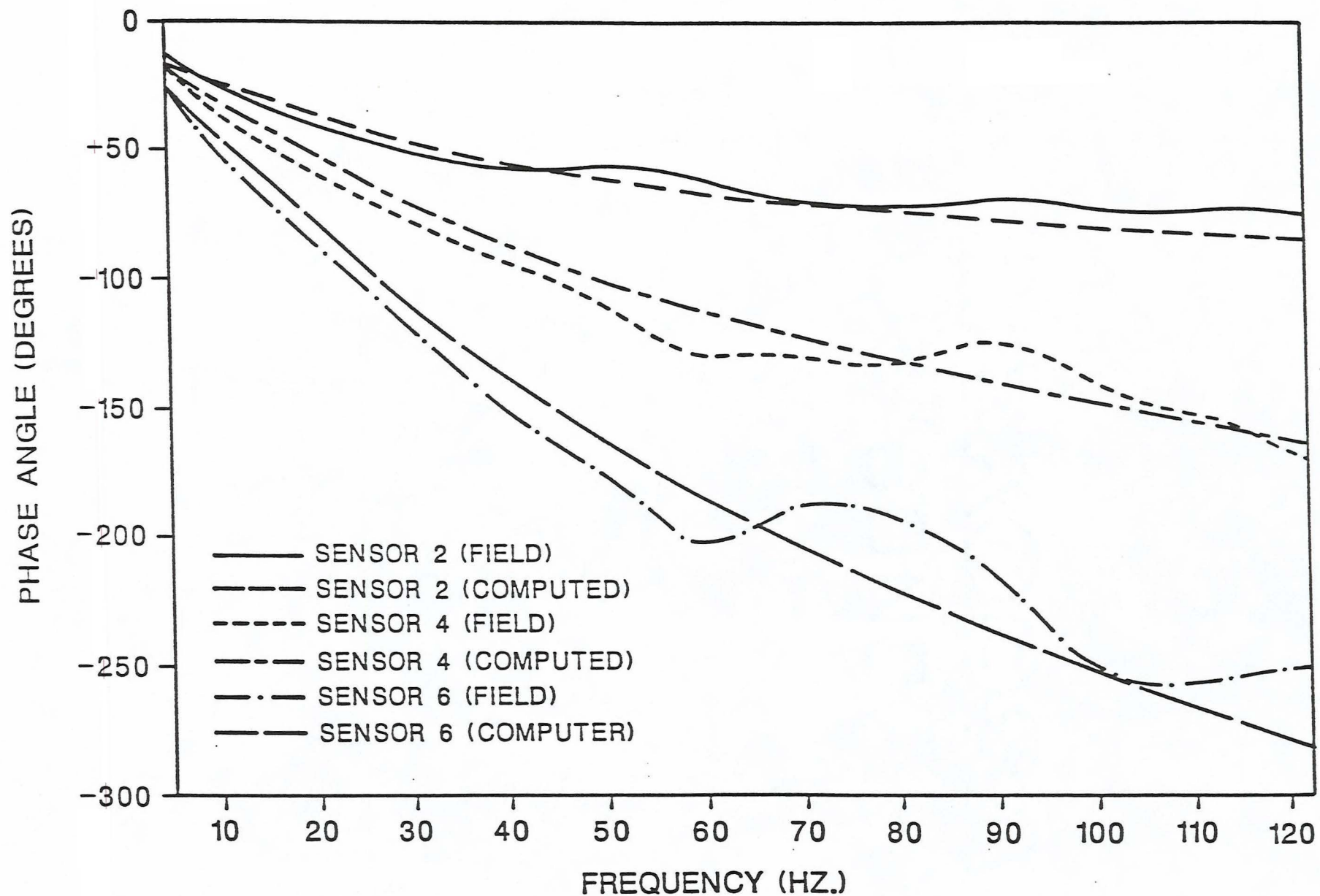


Figure 16. Graph of the phase angle of the transfer function versus frequency for geophones at radii 1, 3, and 5 feet (0.3, 0.9, and 1.5 m.) (Sensors 2, 4, and 6.)

from the center of the loaded area. These are marked on Figures 15 as sensors 2, 4, and 6, respectively. The magnitude is the deflection per unit of force at each frequency and the phase angle represents the time lag of the response behind the impulse at each frequency.

The phase angle is made up of two parts: 1) the time lag due to the propagation of the waves from the load which is called "radiatedamping", and b) the time lag due to material damping of the waves. It is impossible to separate the two kinds of damping without the use of a computer program which calculates the wave motion and represents the material property of each layer as a complex modulus, E^* , which is itself a complex number that is a function of frequency. Several of such programs exist, including the PUNCH program by Kausel and Peek (31), the UTFWIBM program by Roesset (32), and the SCALPOT program by Magnuson (33). In each program the complex modulus of each layer is made up of a real and imaginary part, each of which may be a function of frequency.

$$E^*(f) = E'(f) + iE''(f) \quad (12)$$

where: $E^*(f)$ = the complex modulus,
 $E'(f)$ = the real part of the complex modulus, which is the in-phase component of stress divided by the strain,
 $E''(f)$ = the imaginary part of the complex modulus which is made up of the lagging component of the stress divided by the strain, and
 i = $\sqrt{-1}$

Another way of representing the complex modulus is by using the percent material damping, β .

$$E^* (f) = E' (f) [1 + i2\beta] \quad 913)$$

where: β = the percent material damping, expressed as a decimal,
and

$$2\beta = \tan \phi (f) = \frac{E''(f)}{E'(f)} \quad (14)$$

where: $\phi(f)$ = the material damping lag angle.

Using a process that is exactly analogous to the backcalculation of layer moduli, it is possible to use one of the programs mentioned previously, to assume an E' and a β for each layer and by trial-and-error match the magnitude and phase angle for each of several frequencies and thus to determine the complex modulus for each layer as a function of frequency. In Figures 15 and 16, the computed values of material damping in each of the four layers were as follows: asphaltic concrete, 20%; crushed limestone base course, 4.7%; and clay subgrade, 7.5%. Several trials were made before arriving at these values of damping. A good match of **both** the magnitude and phase angle graphs for all of the sensors and for a number of frequencies (at least four or five) will indicate that convergence on all E' and β values has been achieved. Obviously, this trial and error procedure requires many more trial calculations than with the elastic case, making an efficient search technique and the use of data base methods more imperative.

As an indicator of the size of the material damping to be expected in asphaltic concrete, Figure 17 shows the magnitude and material damping phase angle at 77°F (25°C). This graph was taken from a dissertation by Papazian in 1961 (34). The material phase angle varies between 10° and 63°, corresponding to a percent damping, β , of 9% to 98%. The percent damping varies with the frequency of loading. The results reported by Papazian were determined with creep tests rather than with cyclic load tests and thus the compliance of the testing equipment has little effect on the results.

A general method of converting creep test results into complex moduli has been developed by Pipkin (35), and is summarized by the equation:

$$D^*(\omega) = [\tilde{D}(s)]_{s \rightarrow i\omega} \quad (15)$$

where: $\tilde{D}(s)$ = the Carson transform of the creep compliance, $D(t)$,
 $D^*(\omega)$ = the complex compliance of the material which is a function of the frequency, $\omega (= 2\pi f)$, and

$$D^*(\omega) = D'(\omega) - iD''(\omega) \quad (16)$$

Thus, for a power law creep compliance,

$$D(t) = D_0 + D_1 t^m \quad (17)$$

the Carson transform of $D(t)$ is:

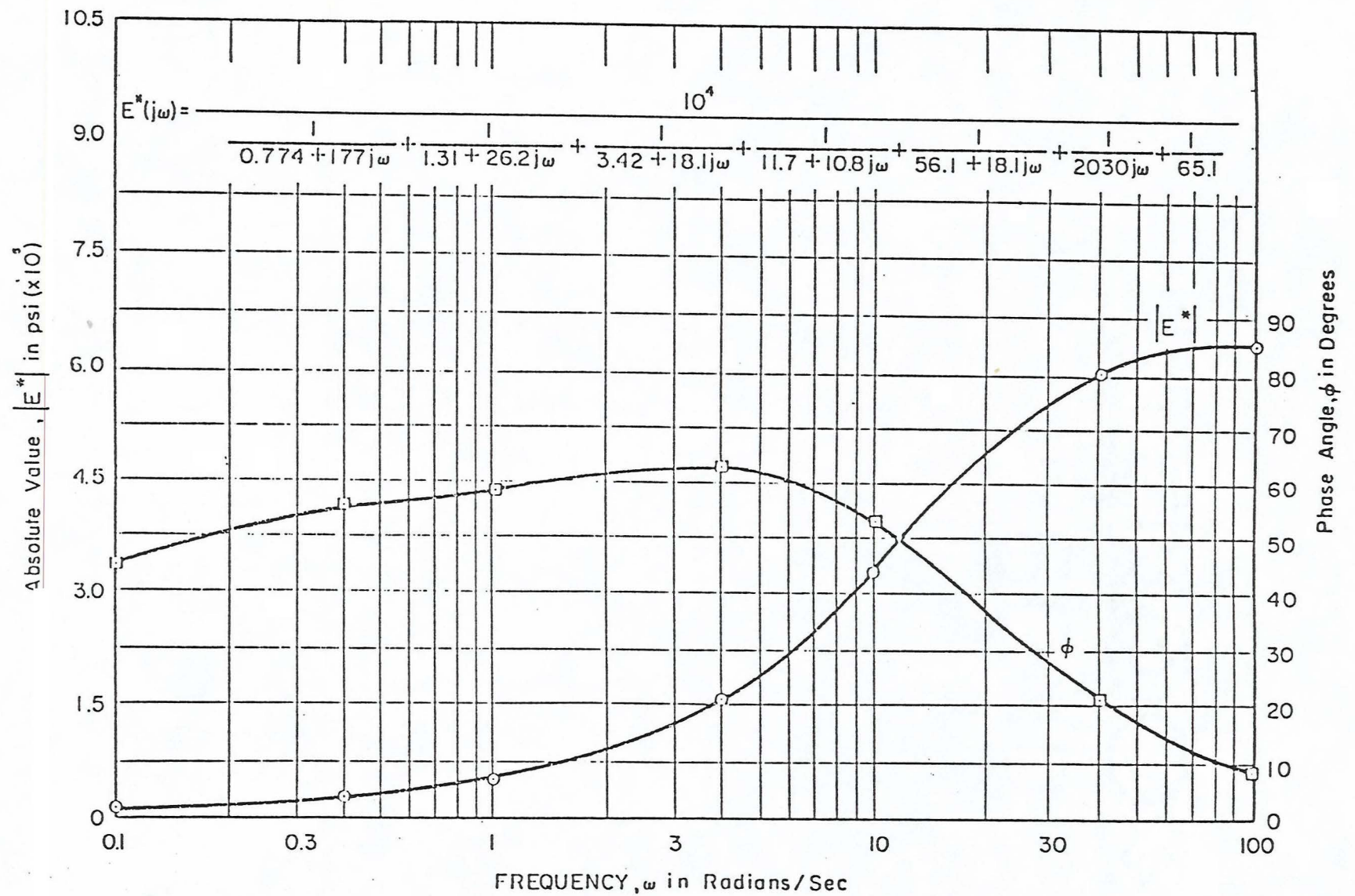


Figure 17. Magnitude and phase angle of the complex modulus of asphalt concrete at 77 F (25 C) (Ref. 32).
(1 psi = 6.895 kPa)

$$\Delta(\sigma) = \Delta_0 + D_1 \Gamma(1+m) s^{-m}$$

When $s = i\omega$ is substituted into the above expression, $D^*(\omega)$ is the result.

$$D^*(\omega) = D'(\omega) - iD''(\omega) \quad (16)$$

where:

$$D'(\omega) = D_0 + D_1 \Gamma(1+m) \omega^{-m} \cos\left(\frac{m\pi}{2}\right) \quad (19)$$

and

$$D''(\omega) = D_1 \Gamma(1+m) \omega^{-m} \sin\left(\frac{m\pi}{2}\right) \quad (20)$$

The percent material damping, β , is given by:

$$\beta = \frac{1}{2} \frac{D''(\omega)}{D'(\omega)} \quad (21)$$

which will vary with frequency, ω , and the material damping phase angle, $\phi(\omega)$, is approximately:

$$\phi(\omega) \cong \frac{m\pi}{2} \quad (22)$$

where D_0 is much smaller than D_1 .

Thus, the slope of the log creep compliance versus log time curve, m , is a direct indication of the material damping phase angle. The complex modulus is directly related to the complex compliance by:

$$E^*(\omega) = \frac{1}{D^*(\omega)} \quad (23)$$

and can be determined frequency by frequency.

The importance of the log-log slope, m , cannot be exaggerated. There is a well-established relationship between the m_t from a tensile creep test and the fatigue exponent, K_2 , as follows:

$$K_2 = \frac{2}{m_t} \quad (24).$$

Speculative derivations concerning compressive creep and recovery tests indicate that the power, α , in the permanent deformation function, $F(N)$, which is used in the VESYS-programs (36):

$$F(N) = \mu N^{-\alpha} \quad (25)$$

is also related to the m -value in the compressive creep curve.

$$\alpha = 1 - m_c \quad (26).$$

The relation between the compressive and tensile values of the log-log slope, m_c and m_t , may follow the rule of mixtures in which:

$$m_c = m_s \theta_s + m_t \theta_a \quad (27)$$

where: θ_s, θ_a = the volumetric content of the solids and the asphalt, respectively, and

m_s = the log-log slope of the creep compliance of the aggregate.

Table 5 gives typical values of the compressive m_c -values of base course and subgrade soils, the percent damping, material phase angle, unit weight, water contents, and volumetric contents of aggregates and water. The values of m_c in the table are actual values, and the damping phase angle and percent damping are calculated from these m_c -values using the approximate relation between m_c and the damping phase angle, ϕ , given in Equation 22. All of the m_c -values can be calculated using the rule of mixtures given in Equation 27, assuming that the m -values for aggregate and water are 0.02 and 0.25, respectively.

Whether this speculation concerning the relationship between the compressive and tensile m -values and the rule of mixtures proves to be definitive or not remains for future research to determine. The fact that Table 5 shows that such a relationship produces consistent values of m gives reason for optimism. The possibility that m -values are related to both the fatigue cracking and rutting properties of materials makes this search much more important to pursue. In general, the dynamic analysis of the full impulse and response signal measured on a pavement in the near field shows promise of providing in situ material properties of each of the pavement layers which include not only the frequency dependent complex moduli in the range of frequencies generated by traffic loadings but also material properties that are related to fatigue cracking and permanent deformation. Because of this, dynamic analysis is decidedly a subject to develop further in the future.

Table 5. Table of Typical m-Values and Percents Damping for Base Course and Subgrade Materials

Material	M_c	Percent Damping β	Damping Lag Angle ϕ	Unit Weight lb/ft ³ *	Typical Water Content	Volumetric Aggregate Content θ_s	Volumetric Water Content θ_w
Gravel	0.04	1.6	1.8	130	0.05	0.75	0.10
Silt	0.06	4.7	5.4	110	0.12	0.59	0.19
CL Clay	0.08	6.3	7.2	115	0.18	0.59	0.28
CH Clay @ Plastic Limit	0.096	7.5	8.6	130	0.20	0.60	0.35
CH Clay @ Liquid Limit	0.14	11.2	12.3	105	0.50	0.42	0.56

Assumptions: m_s = 0.02 (aggregate)
 m_w = 0.25 (water)
 G_s = 2.65 Specific Gravity of Solids

* 1 lb./ft³ = 0.158 kN/m³

Systems Identification Methods

"Systems Identification" methods were developed by electrical engineers who were interested in determining the characteristics of a filter by using the input and output signals of the filter and an assumed model of the filter (37). The characteristics of the model are changed systematically using a search technique until the model produces an output that is acceptably close to that of the filter. This procedure is, in fact, exactly analogous to what is being done in backcalculating the moduli of pavements. However, because a complete input and output signal is analyzed, the process is identical with dynamic analysis. This is fortunate because standard public domain software has been developed to perform the analyses to determine the filter characteristics.

The same methods can be applied to the analysis of surface wave spectra, as will be illustrated below. Surface waves are generally high frequency, far field measurements that are made outside of the deflection basin. An impulse load is applied to the pavement at a remote distance and the surface motion is sensed as signals received by two sensors such as geophones, which are separated by a distance, x . The pavement between the two sensors is regarded as the filter, the characteristics of which are to be determined. The unknowns in each layer are its modulus, percent damping, thickness, and unit weight. To the extent that any of these can be assumed, the search will be more simple. The Fast Fourier Transform is applied to each of the two signals to break them down into their component frequencies.

~~The first step is to select a model for the pavement using a program~~ such as the previously mentioned PUNCH, UTFWIBM, or SCALPOT programs.

Next is to set all of the known values such as the layer thickness and unit weight of the materials in each layer.

The third step is to assume realistic values of the modulus and damping of each layer as they vary with frequency. These as yet unknown variables are denoted as E_j for the purposes of the subsequent discussion. In the case of a four-layer pavement, the index, j , would range from 1 to 8, to account for both the modulus and the damping in each layer.

The fourth step is to determine a sensitivity matrix for the model, $F_{i,j}$, which is composed of the relative change in the numerical partial derivative of the calculated surface wave velocity of the model, V_{mi} , for each frequency, f_i , with respect to each layer variable, E_j . An approximate central difference formula for this partial derivative is:

$$\frac{\partial V_{mi}}{\partial E_j} = \frac{V_{mi} [E_j + \Delta E_j] - V_{mi} [E_j - \Delta E_j]}{2 \cdot \Delta E_j} \quad (28)$$

where: ΔE_j = a prescribed change in the value of each layer variable, E_j .

The relative change of this partial derivative is the desired term in the sensitivity matrix:

$$F_{i,j} = \frac{E_j}{V_{mi}(E_j)} \cdot \frac{\partial V_{mi}}{\partial E_j} \quad (29)$$

If there are n frequencies and p layers, the sensitivity matrix, F , will have n rows and $2p$ columns.

The fifth step is to compute from the field data the pavement surface wave velocity of each frequency, V_{pi} , for frequency f_i , in Hertz.

This is determined by getting the difference in phase lag between the two sensors, $\Delta\phi_{p1}$, and using the formula:

$$V_{p1} = \frac{x \cdot 360f_1}{\Delta\phi_{p1}} \quad (30).$$

In a similar way, a pavement modeled using programs PUNCH, UTFWIBM, or SCALPOT can be excited at frequency, f_1 , and the calculated phase lag from the model, $\Delta\phi_{m1}$, can be determined for each frequency, and from this, the surface wave velocity of the model, V_{m1} , can be determined and a relative error in the velocity in the k^{th} iteration can be found, and defined as the residual error, $(r_i)_k$:

$$(r_i)_k = \left[\frac{\Delta V_i}{V_{mi}} \right]_k = (\Delta\phi_{mi})_k \left[\frac{1}{\Delta\phi_{p1}} - \frac{1}{(\Delta\phi_{mi})_k} \right] \quad (31)$$

The relative changes in the layer properties, for the k^{th} iteration is denoted as $(d_j)_k$ and is given by:

$$(d_j)_k = \frac{(E_j)_{k+1} - (E_j)_k}{(E_j)_k} \quad (32)$$

The relationship between the sensitivity matrix, F_{ij} , the residual error, $(r_i)_k$, and the relative change in layer properties, $(d_j)_k$, is given in matrix form as:

$$F \tilde{d}_k = \tilde{r}_k \quad (33)$$

The size of these matrices is: F , (n rows by $2p$ columns),; \tilde{d}_k , ($2p$ rows by 1 column); and \tilde{r}_k , (n rows by 1 column), where n is the number of frequencies and p is the number of pavement layers.

The unknown relative change in the layer properties is determined with each iteration from:

$$F^T F \tilde{d}_k = F^T \tilde{r}_k \quad (34)$$

by multiplying both sides of Equation (34) by the inverse of $F^T F$. It is noteworthy that Equation (34) is identical with Equation (4) which was used to minimize the squared errors between the calculated and observed deflections of a layered elastic pavement. Thus, the search techniques are identical, and Equation (34) is a generalization of the elastic case. New estimates of the phase difference, $\Delta\phi_{mi,k+1}$ for the $(k+1)^{st}$ iteration can be determined from the numerical partial derivatives and the terms in the sensitivity matrix, neither of which change with iterations. The equation for the new estimate of the phase difference is:

$$(\Delta\phi_{mi})_{k+1} = \frac{F_{i,j,k+1}}{\left[\frac{\partial V_{mi}}{\partial E_j} \right]} \cdot \frac{360 f_i x}{(E_j)_{k+1}} \quad (35)$$

Analyses of this sort may be made with any procedure for determining layer properties, even including the thickness of the layer as one of the unknown variables. The results of surface wave analysis, being a far field analysis, will be for frequencies that are much higher than those caused by pavement loadings and thus a frequency correction must be

developed and applied to the final material properties to which the search process converges.

Systems identification methods require a model to be identified, reasonable starting values of the unknown variables to be assumed, and a set of sensitivity coefficients and numerical partial derivatives to be determined before the analysis begins. The sensitivity matrix and partial derivatives remain unchanged thereafter. They can be used for layered elastic analyses, finite element analysis, dynamic analysis of the near field and surface wave analysis of the far field. Algorithms already exist which can exercise the selected model to develop the sensitivity matrix, F , and to step through the iterative search until final convergence is reached on all unknown variables while satisfying a criterion to minimize the sum of squared relative errors. Further development and use of systems identification methods in the backcalculation of layer properties is warranted in the future.

USE OF NONDESTRUCTIVE TESTING IN THE FUTURE

The use of nondestructive testing devices may be only beginning to be developed toward its full potential. Recording the full impulse and response signals and the use of dynamic analysis suggests that in the future more properties of materials can be extracted from nondestructive testing in the field than are measured at present. Recording and processing information from multiple sensors on the same vehicle suggests the possibility that many, if not all, of the material properties of each layer, may be measured nondestructively. This includes the constitutive relations, and the properties that govern the fatigue cracking of the asphaltic concrete and the permanent deformation of all of the layers.

The speculation may not be too remote from reality that radar may be used to determine layer thickness and asphalt content if the pavement layers are viewed as filters and systems identification methods are used to determine the three unknowns in each layer: the thickness, the volumetric concentration of the aggregate and the liquid, either asphalt or water. This assumes that the dielectric constant of a layer obeys a rule of mixtures in combining the dielectric constants of the solid and liquid constituents of the layer.

In addition to layer moduli, we may look forward to backcalculating the creep compliance of each layer with time and its complex modulus as it depends upon frequency directly from nondestructive testing results. Fracture properties of the bound layers and the permanent deformation properties of each layer may also be susceptible to backcalculation, insofar as they are related to the creep and relaxation of the material. Residual stresses, which are imparted to each pavement layer by compaction and by each passing traffic load, actually prestress a pavement layer and make it possible even for a granular, unstabilized layer to carry tensile stresses. Means of estimating the residual stresses from field nondestructive test measurements are highly desirable. It is speculation once more, but the residual stresses may be related to or at least able to be calculated from the permanent deformation properties of all layers (38,39).

Systems identification methods offer an attractive, developed methodology to use in backcalculating layer moduli, material damping percentages, and even layer thicknesses. ~~The material properties thus backcalculated~~ may even include the frequency dependence of the modulus and material damping. Because these methods satisfy a least squared

relative error criterion which is consistent with manufacturer's specifications of sensor precision, these methods may prove to be a useful and efficient method of searching for the unknown variables in each pavement layer, and may provide a standard way to reduce NDT data.

The errors that are inherent in the backcalculation of layer properties include both systematic errors introduced by the analysis model and the assumed constitutive relations and random errors due to load and displacement measurements and spatial variations of the material properties. Future work on improving nondestructive testing measurements must concentrate upon reducing or eliminating the random errors by repeated measurements and more precise instruments, as well as upon reducing the systematic errors by using more realistic pavement models and constitutive relations.

The search methods that are used to converge on layer properties will undoubtedly need to be aided by an expert analyst or an expert system, particularly when the objective is to determine the components of layer complex moduli as they vary with frequencies. Some interesting observations that are related to this are the implications to be drawn from the fact that Equations (4) and (34) are identical. Equation (4) is the matrix equation for the search for elastic layer moduli from surface deflections and Equation (34) is the matrix equation for the search for layer moduli and damping from surface wave speeds. Mathematical proof was provided that Equation (4) converges to a global minimum rather than a local minimum in the error surface and by inference, Equation (34) should do so, also. Furthermore, because the search methods are identical, neither method has a superior claim to achieve a unique solution to a set of layer moduli. However, as noted previously, and illustrated in Figure

3, it is possible to obtain two or more correct solutions, only one of which represents acceptable values of layer moduli. Once more, this dilemma is solved by the intervention of an expert.

CONCLUSION

A few concluding observations are in order. First of all, the uses of nondestructive testing have not reached an end point. Rather, it is only at the beginning of its potential usefulness. More precision is needed from the testing equipment and procedures, and more realistic models and constitutive relations will reduce the size of systematic errors. Expert analysts or expert systems need to control the backcalculation procedures in order to assure that acceptable results are achieved. Multiple sensors used in tandem may be in the not-too-distant future of nondestructive testing. For example, radar and impulse-and-response near field testing coupled with modern systems identification techniques show promise of providing layer thickness, asphalt or water content, moduli, damping, creep and relaxation, and fracture and permanent deformation properties of pavement layers in the field. This will make it possible to predict the remaining life of a pavement in the field immediately after it has been tested. It will also make it possible to use this kind of multiple sensor testing to perform specification testing.

A strategic objective of nondestructive testing should be to become the primary method of measuring pavement material properties. When this objective is reached, laboratory measurements may become the secondary ~~method of materials testing which are used with new materials and for~~ confirming field test results.

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